

# Midwinter meeting in discrete probability

## January 14-15, 2026, Umeå

All talks are in room MIT.C.413 in the MIT-building.

Each speaker has either a 35 minute time slot followed by five minutes for questions and time for the next speaker to prepare.

### Wednesday

#### 9.00 Welcome

#### 9.00-9.35 He Guo: Semi-random greedy independent set algorithm

We study a semi-random (Rodl nibble) variant of the random greedy independent set algorithm to construct a large independent set in a hypergraph. Let  $r \geq 2$  be a fixed integer and let  $\mathcal{H}$  be a  $r$ -uniform  $N$ -vertex hypergraph satisfying that each vertex is contained in at most  $D$  edges. Assuming  $\min\{N^{r-1}/D, D\} \geq f$  for some  $f \geq (\log N)^c$  with  $c > 0$ , we prove that if  $\mathcal{H}$  satisfies some degree and codegree conditions, then with high probability there are  $\Omega\left(N\left(\frac{\log f}{D}\right)^{\frac{1}{r-1}}\right)$  vertices in the independent set constructed by the algorithm.

A key improvement of this semi-random variant compared to Bennett and Bohman's previous random greedy version is that we replace their requirement of  $\mathcal{H}$  being  $D$ -regular by the maximum degree condition. We also prove that the independent set constructed by this algorithm has various pseudo-random properties, including those needed for applications in Ramsey theory. And we prove that the same holds for a random subhypergraph, i.e., the above properties remain true in the sparse setting.

Based on joint work with Lutz Warnke.

#### 9.40-10.15 Svante Janson: Simply generated trees conditioned on the number of vertices of given degrees.

It is well known that simply generated trees with a given number of vertices include Galton-Watson trees conditioned on a given number of vertices, and that simply generated trees often, but not always, can be reduced to the Galton-Watson case by tilting. This extends to trees with a given number of leaves, or more generally a given number of vertices with degrees in some given set. The Galton-Watson case has been studied by several authors, including Rizzolo,

Kortchemski, Abraham & Delmas. I will discuss extensions of their results and methods, in particular tilting, to the simply generated setting.

## **10.20-10.40 Coffee**

### **10.40-11.15 Fredrik Scheie : Sequential acquisition process for finite agents**

We here introduce a population model called the sequential acquisition model (SAM), a neutral model of cultural evolution formulated as a time-homogeneous Markov jump process where a finite population of  $N$  agents sequentially invent, acquire, and forget traits in continuous time. The aim of this talk will be to discuss properties of the SAM model. We here analyze the resulting trait-repertoire dynamics and show that SAM admits a unique limiting distribution. From this distribution, we can derive explicit expressions for the expected number of traits at equilibrium, which depend among others on a parameter  $\alpha$ , describing the rate of the socially transmitted traits through an acquisition process. We further obtain results for the expected number of traits carried per agent and characterize the duration of the accumulation process. Together, these results provide a tractable baseline for studying sequential cultural transmission where we hope future population models may further build upon this work. This is a joint work with Fredrik Jansson and Jonas Sjöstrand.

### **11.20-11.55 Amin Coja-Oghlan: The Ising model on random graphs at low temperature**

We revisit the Ising model on sparse Erdos-Renyi random graphs. While a first order approximation to the free energy density was obtained by Dembo and Montanari (2010), here we aim to identify the precise limiting distribution of this random variable. Specifically, we establish a central limit theorem in the low temperature regime. Along the way we clarify the 'pure states' of the model, a question that had not been answered satisfactorily in prior work.

## **Lunch 12.00-13.15**

### **13.15-13.50 Pavel Zakharov: The Ising model on random graphs: high temperature behaviour.**

The Ising model provides one of the clearest examples of a nontrivial phase transition. Lyons identified the existence of such transition on trees and tree-like graphs. We show that, in the absence of an external magnetic field, the free energy of the Ising model on Erdos-Renyi random graphs superconcentrates, as opposed to the low-temperature regime.

### **13.55–14.30 Julien Verges: Lower-tail large deviations in first-passage percolation**

Endow every edge of the usual  $Z^2$  lattice with a random positive weights, and consider the associated random metric  $T$ . It is known that almost surely, seen from far away,  $T$  looks like some deterministic norm. In this talk I will try to

explain can we go further and give a (rough) estimate of the probability that  $T$  looks like some other metric instead.

#### **14.30-15.00 Coffee**

#### **15.00-15.35 Victor Dubach: Scaling limits of descent-biased trees**

The  $q$ -descent-biased distribution is a probability distribution on rooted labelled trees introduced recently by Paul Thévenin and Stephan Wagner. Therein, the probability mass of a given tree  $t$  is proportional to  $q^{des(t)}$ , where  $des(t)$  is the number of descents in the tree  $t$ . In their paper, PT and SW proved that this random tree  $T_n^{(q)}$  admits an explicit "one-ended" local limit at the root when  $q$  is fixed (i.e., independent of  $n$ ).

A complementary question is that of the global limit (also known as scaling limit) of  $T_n^{(q)}$ . Our main result is a phase transition when  $q$  depends on  $n$  and is of the form  $q = a/n$  for some fixed  $a > 0$ . In that regime, we establish a scaling limit for  $T_n^{(q)}$  in the framework of dendrons:  $T_n^{(q)}$  converges in distribution to a non-trivial random dendron that depends on  $a$ . When  $a = 0$ , we retrieve the trivial dendron limit of recursive trees. In addition, when  $q$  is fixed, we prove that  $T_n^{(q)}$  admits the Brownian Continuum Random Tree as its scaling limit, in the sense of the Gromov-weak topology (which coincides with the topology of dendrons). This was famously proved by Aldous for Cayley trees (i.e.,  $q = 1$ ).

Joint work with Paul Thévenin and Stephan Wagner.

#### **15.40-16.15 Tom Britton: Stochastic epidemic models with group structures**

We consider a stochastic epidemic model in a community structured into small groups, having global as well as local contacts. Focus lies on comparing the probability, as well as the size, of a major outbreak, of the model with a situation that is closer to having only global contacts (homogeneous mixing). We move towards homogeneous mixing either by swapping some of the local contacts to global contacts, or by making the group sizes bigger. Joint work with Frank Ball and Peter Neal.

#### **18.30 Dinner at *Bröd och Vin* for registered participants**

## Thursday

### 9.00-9.35 Ruoyu Wang: Local convergence and subtrees of trees

In this article, we study the asymptotic behavior of the number of subtrees and subtree density for a sequence of trees that converges in the Benjamini–Schramm sense. Benjamini–Schramm convergence, also called local (weak) convergence, describes the local behavior of a sequence of graphs. Here we show that for a sequence of Benjamini–Schramm-convergent trees, the average subtree entropy, i.e., the logarithm of the number of subtrees divided by the order, converges to a constant depending only on the limit. Furthermore, we show that if a sequence of trees locally converges to a limit that is not a two-way infinite path, then the subtree density, i.e., the probability of a uniformly random vertex being contained in a uniformly random subtree, also converges to a constant that depends only on the limit.

### 9.40-10.15 Anda Skeja: Multivariate Dependencies in Multiplex Graphons

Pairwise interlayer dependence can miss genuinely multivariate structure in random multiplex graphs. A simple example is XOR: generate two layers independently and define a third as their edgewise XOR, then the first two layers look independent marginally, yet become dependent once the third layer is known.

We study this problem in the well-studied class of exchangeable multiplex graphs, which encompasses many random graph models. We introduce a family of relabeling-invariant, graphon-level information functionals built around the *joint graphon entropy*, which capture higher-order dependence and detect XOR-type synergy. We also discuss a nonparametric approach to estimating these quantities from a single observed  $n$ -vertex multiplex under standard smoothness assumptions. In the single-graph (single-layer) setting, we briefly discuss limiting distributions for the graphon entropy estimator in related random graph models. Based on joint work with Sofia Olhede.

### 10.20-10.40 Coffee

### 10.40-11.15 Stephan Wagner: Continuous parking sequences

Classical parking functions are defined by the following parking procedure:  $n$  consecutive parking spots are available on a street. There are  $n$  cars with preferred parking spots  $\pi_1, \pi_2, \dots, \pi_n$ . Now the  $i$ -th car that arrives parks in spot  $\pi_i$  if it is available, and otherwise in the first empty spot that comes afterwards. If the cars can successfully park according to this procedure, then  $\pi_1, \pi_2, \dots, \pi_n$  is called a parking sequence.

We introduce the notion of continuous parking sequences as a continuous analogue of parking functions, where the lengths of the cars and the street can be arbitrary positive real numbers. The set of parking sequences is a union of polytopes whose volume is given by a surprisingly simple product formula. The proof of this formula is based on a certain convolution identity. Statistical

properties of parking sequences chosen uniformly at random are discussed as well, especially concerning the first and last car in the sequence.

### **11.20-11.55 Lorents Landgren: Maximizing the number of connected spanning subgraphs**

Starting with the set of connected multigraphs with  $n$  vertices and  $m$  edges, we define  $c_j(G)$  to be the number of connected spanning subgraphs of  $G$  with exactly  $j$  edges. These numbers show up as coefficients of the reliability polynomial  $R(G, p)$ , which is the probability that  $G$  remains connected after each edge has been independently retained with probability  $p$  and otherwise deleted. It is therefore natural to ask if a given graph  $G$  can be modified to increase the  $c_j$ 's, and to try to identify the graphs that maximize these respective values.

General answers to these puzzles are known only for the cases where just one or two edges are removed, i.e. for  $j$  equal to  $m - 1$  and  $m - 2$ . There is also a claimed but seriously flawed solution for  $j = m - 3$ . We present new solutions for the two latter cases. In particular, we show how the  $c_{m-3}$ -maximizing graphs can easily be constructed from the set of cubic graphs on  $2(m - n)$  vertices that satisfy certain natural connectivity properties.

### **Lunch 12.00-13.15**

### **13.15-13.50 Jacob Lundblad: The terminal Wiener index of random trees**

The Wiener index, defined as the sum of all pairwise distances between vertices in a tree, is a well studied property for many common types of random trees. In 2009 the Terminal Wiener index was introduced by Gutman, Furtula and Petrović, motivated by potential applications in chemistry. To obtain the Terminal Wiener index of a rooted tree, for which few previous results exists, one considers instead the sum of pairwise distances between leaves. In this talk we show that for certain types of increasing trees, the difference between the average distance between random vertices and the average distance between leaves is small, corresponding to the behaviour of the Terminal- and regular Wiener indices.

### **13.55-14.30 Jasper Ischebeck: Apollonian networks and $k$ -heavy trees**

An Apollonian network is an iteratively generated planar graph that is in bijection with ternary trees. The problem of finding the longest path in an Apollonian network can be bounded by finding a large binary subtree of this ternary tree. A greedy approach from the top that chooses the 2 largest branches is called the 2-heavy tree.

The Apollonian network / ternary tree can be generated by various random models, e.g. conditioned Galton-Watson trees and split trees. For conditioned Galton-Watson trees, it has been shown that the 2-heavy tree has  $O(n)$  nodes and therefore covers a non-vanishing fraction of the tree. In this talk, we focus on split trees, where we show that the 2-heavy tree has  $\Theta(n^\beta)$  nodes, with some

$\beta \in (0, 1)$  given as a solution of an equation. With the contraction method, a limit distribution can also be found.

#### **14.35-15.10 Altar Ciceksiz: Negative Correlation for Random Cluster Model and Uniform Forest Measure**

We show that negative correlation for forests in the complete graph holds in the large  $\lambda$  limit. We also show that the random cluster model on the complete graph for  $q \geq 1$  is negative correlated when  $\lambda$  is at least one. These are the first non-trivial results for this problem for this important special case

#### **15.15-15.30 Coffee**

#### **15.30– Fiona Skerman: Testing between planted distributions and connections to recovery**

While there has been much work on studying the low degree polynomial framework for measuring the computational hardness of problems when one wishes to detect the presence of hidden structures, a recent line of work [Schramm-Wein 2022] has extended the framework to the setting where one wishes to recover a planted structure from noisy data. We use this recent work to show a connection between detection and recovery.

In particular, we establish that for any given recovery problem where one wishes to estimate some scalar quantity of the planted distribution from some observation, there is a specific testing problem with the same computational hardness regimes. In the other direction, for any testing problem so that the likelihood ratio of the signal exists, there is a corresponding recovery problem with the same computational hardness regimes.

Joint work with: Cynthia Rush, Alex Wein and Dana Yang.