Internship Programme

Project list for the calendar year 2027

Department of Mathematics and Mathematical Statistics
Umeå University, Umeå, Sweden

About the programme

Our internship programme is a one-term opportunity based in Umeå, where students work directly with researchers in their chosen field. The programme provides a unique chance to bridge the gap between academic studies and their application in a research environment.

Eligibility and Applications

The programme is primarily aimed at graduate students from our partner universities, but applications from students at other universities are also welcome. Application rounds are held twice a year.

Autumn round: September 15 – October 15

Students apply for **one** specific project that best matches their academic background and research interests. Applications are submitted via our online form and must include:

- Curriculum Vitae (CV)
- Transcript of records (in English)
- Motivation letter

Support Provided

- Coverage of round-trip travel to Umeå (economy class, funded by the department).
- Assistance with securing student accommodation (costs covered by the intern).
- Support in applying for an Erasmus+ stipend through the home university.
- Access to a supportive work environment at Umeå University, including a workspace, necessary materials, and insurance.

Selection Procedure

Applications are reviewed by a working group in collaboration with the prospective supervisors. Candidates are selected based on eligibility, academic record, and compatibility with the chosen project.

Content of the Document

This document provides information about the projects available in the current application round, along with the corresponding supervisors. Applicants must select one project to apply for. The projects are organised by research area and supervisor.

For more information and access to the application form, please visit:

https://www.umu.se/en/department-of-mathematics-and-mathematical-statistics/education/internship-programme/

Discrete Mathematics

Victor Falgas Ravry

Victor Falgas Ravry is an Associate Professor in Mathematics. His main research interests lie in extremal combinatorics and discrete probability — interests which are shared by several others members of the Discrete Mathematics group at Umeå. Typical topics include extremal graph theory, Ramsey theory, games on graphs and random graph models.

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Project 1: Maker–Breaker percolation games

Maker–Breaker games were introduced in an influential paper of Chvátal and Erdős [2] in the late 1970s and form a rich and widely studied class of positional games.

The board in a Maker–Breaker game consists of a set B (for example, the set of edges of a graph, or the cells of a tic-tac-toe grid). Two players, known as Maker and Breaker, take turns claiming elements of the board B. Maker's goal is to claim all elements of some member W of a pre-specified family $\mathscr{W} \subseteq \mathscr{P}(B)$ of winning sets (for example, a winning set could be all the edges of a triangle, or three squares on the same row, column or diagonal of a tic-tac-toe grid). Breaker's aim is to prevent this from happening by claiming at least one element from each $W \in \mathscr{W}$.

As the vague and general description above suggests, there are many games within this class, and many questions one could ask: who has a winning strategy under optimal play? how long will the game last? what happens if one of the players selects his or her moves at random? what happens if we allow one of the players to claim more elements of the board per turn than the other player?

A rich theory has developed in a bid to answer these questions (see e.g. the book of

Beck [1] and the survey of Stojaković and Szabó [8]), and to elucidate some intriguing connections between these games and deep phenomena in discrete probability and Ramsey theory. As Tibor Szabó has pointed out, these delightfully fun and deceptively light-hearted combinatorial games have contributed to many of the most significant advances in discrete mathematics in the past half century, and are well worth studying for their own sake.

A few years ago, I introduced a new class of Maker–Breaker games with motivation coming from percolation theory. These games are played on a (large) graph G with two specified vertices u and v. Maker and Breaker take turns claiming sets of m and b edges of G respectively, and Maker's aim is to build a path joining u to v.

These Maker–Breaker percolation games may be viewed as generalisations of the classical Shannon switching game (which corresponds to the special case $m = b = 1$, and was fully resolved by Lehman [7] in the 1970s), and have been the subject of two articles by my postdoc Nicholas Day and myself [4, 3], and more recently to two lovely papers of Dvořák, Mond and Souza [5]. As can be seen from a cursory glance at the concluding sections of these four papers, a great many open questions remain — many of which would be highly suitable for an internship, including in particular the as-yet unstudied site percolation version of the game (where Maker and Breaker claim vertices rather than edges of G).

References

[1] József Beck. *Combinatorial games: tic-tac-toe theory*, volume 114. Cambridge University Press Cambridge, 2008.

[2] Vašek Chvátal and Paul Erdős. Biased positional games. Pp. 221–229 in *Annals of Discrete Mathematics*, volume 2. Elsevier, 1978.

[3] A. Nicholas Day and Victor Falgas-Ravry. Maker–breaker percolation games I: crossing grids. *Combinatorics, Probability and Computing*, **30**(2): 200–227, 2021.

[4] A. Nicholas Day and Victor Falgas-Ravry. Maker-breaker percolation games II: escaping to infinity. *Journal of Combinatorial Theory, Series B*, **151**: 482–508, 2021.

[5] Vojtěch Dvořák, Adva Mond, and Victor Souza. The maker-breaker percolation game on the square lattice. To appear in *Journal of Combinatorial Theory, Series B*, 2024.

[6] Vojtěch Dvořák, Adva Mond and Victor Souza. The Maker-Breaker percolation game on a random board. Preprint, arXiv ref: 2402.17547, 2024.

[7] Alfred Lehman. A solution of the Shannon switching game. *Journal of the Society for Industrial and Applied Mathematics* **12**(4): 687–725, 1964.

[8] Miloš Stojaković and Tibor Szabó. Positional games on random graphs. *Random Structures & Algorithms*, **26**(1-2):204–223, 2005.

Project 2: Bootstrap percolation in random intersection graphs

Bootstrap percolation is concerned with the study of a family cellular automata. It first arose in work of Chalupa and Leath and Reich [1] on magnetic phenomena, and has been widely studied as a toy model for the spread of misinformation or of an infection on a network.

Given a graph G, an infection threshold r and a set of initially infected vertices I_0, a bootstrap percolation process on G unfolds as follows: at each time $t \geq 0$, all vertices of G that are adjacent to at least r vertices of the infected set I_t become infected and are added to I_t to form I_{t+1}. The set of eventually infected vertices in this process is denoted by $I_\infty := \bigcup_{t \geq 0} I_t$.

There are many questions one could ask: given an initially infected set I_0 of a given size, how large/how small could the size of the eventual infection $|I_\infty|$ be? How long does the infection take to reach its final state? If the set of initially infected vertices I_0 is selected at random, what is the likely distribution of I_∞? What if the threshold for infection r is not uniform, or if the infection spreads in a random rather than deterministic fashion?

As part of an internship project, I propose to study bootstrap percolation on random intersection graphs, to provide counterparts in that setting to the work of Janson, Łuczak, Turova and Vallier [3] on the Erdős–Rényi random graph model, and of Whittemore [4] and Sarkar and myself [2] on the Gilbert random geometric graph model.

References

[1] John Chalupa, Paul L Leath, and Gary R Reich. Bootstrap percolation on a Bethe lattice. *Journal of Physics C: Solid State Physics*, 12(1):L31, 1979.

[2] Victor Falgas-Ravry and Amites Sarkar. Bootstrap percolation in random geometric graphs. *Advances in Applied Probability*, pages 1–47, 2021.

[3] Svante Janson, Tomasz Łuczak, Tatyana Turova, and Thomas Vallier. Bootstrap percolation on the random graph $G_{n,p}$. 2012.

[4] Alyssa Whittemore. *Bootstrap percolation on random geometric graphs*. PhD thesis, The University of Nebraska-Lincoln, 2021.

Project 3: Mixed degree conditions for hypergraphs

An r-uniform hypergraph (or *r-graph*) is a pair $H = (V, E)$, where $V = V(H)$ is a set of vertices, and $E = E(H)$ is a collection of r-sets from V that form the edges of H. The *degree* $\deg(S)$ of a set S in H is the number of edges of H that contain S. The minimum of $\deg(S)$ over all subsets $S \subseteq V$ of size $|S| = \ell$ is known as the *minimum ℓ-degree* of H, and is denoted by $\delta_\ell(H)$.

A copy of the clique on t vertices $K_t^{(r)}$ in H is a t-set $X \subseteq V$ such that all $\binom{t}{r}$ possibles r-sets from X are present as edges in H. If H contains no such copy, H is said to be $K_t^{(r)}$*-free*.

A classical problem in extremal hypergraph theory is: given integers $1 \leq \ell < r < t$, determine the maximum value of $\delta_\ell(H)$ over all $K_t^{(r)}$-free r-graphs H on $n = |V(H)|$ vertices. The case $r = 2$ was resolved almost a century ago in a celebrated theorem of Turán [5].

For all $r \geq 3$, however, this problem — known as the *hypergraph Turán problem* —- has remained stubbornly open in all cases, and is arguably one of the major open problems in discrete mathematics; see Keevash's survey [1] dedicated to it.

As part of an internship, I proposed to study a mixed-degree version of the hypergraph Turán problem. The focus would be on obtaining good lower bound constructions for this new family of problems, which is as yet unstudied, in order to cast light on the hypergraph Turán problem. Part of the work would build on a mountain of unpublished data on flag algebraic bounds computed by Sliacan, and on a study of the treasure trove of constructions provided by Sidorenko in his 1995 survey [2].

References

[1] Peter Keevash. Hypergraph Turán problems. *Surveys in combinatorics*, 392:83–140, 2011.

[2] Alexander Sidorenko. What we know and what we do not know about Turán numbers. *Graphs and Combinatorics*, 11(2):179–199, 1995.

[3] Paul Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48:436–452, 1941.

Project 4: Covering problems in hypergraphs

An r-uniform hypergraph (or *r-graph*) is a pair $H = (V, E)$, where $V = V(H)$ is a set of vertices, and $E = E(H)$ is a collection of r-sets from V that form the edges of H. The *degree* $\deg(S)$ of a set S in H is the number of edges of H that contain S. The minimum of deg(S) over all subsets $S \subseteq V$ of size $|S| = \ell$ is known as the *minimum ℓ-degree* of H, and is denoted by $\delta_\ell(H)$.

A copy of the clique on t vertices $K_t^{(r)}$ in H is a t-set $X \subseteq V$ such that all $\binom{t}{r}$ possibles r-sets from X are present as edges in H. A $K_t^{(r)}$*-covering* in H is a collection of copies of $K_t^{(r)}$ that together cover all the vertices of H. If these copies may in addition be chosen to be vertex-disjoint (which implies in particular that $|V(H)|$ must be divisible by t), then we say that H contains a $K_t^{(r)}$*-tiling* (also known as a $K_t^{(r)}$*-factor*).

A natural question in extremal hypergraph theory is: given $1 \leq \ell < r < t$, what is the least d such that every r-graph on n vertices with minimum ℓ-degree strictly greater than d must contain a $K_t^{(r)}$-covering? We denote the answer to this question by $c_\ell(n, K_t^{(r)})$, and refer to it as the *ℓ-degree covering threshold* for $K_t^{(r)}$.

The systematic study of covering thresholds was initiated in works of Zhao and myself [2] (in the case $r = 3$, $\ell = 2$) and Markström, Zhao and myself [1] (in the case $r = 3$, $\ell = 1$), and has attracted increasingly significant attention in the last few years. Covering thresholds lie between two of the most studied families of parameters in extremal hypergraph theory, namely the existence thresholds studied in Turán-type problems (so-named after the pioneering work of Turán [5] discussed in the previous subsection) and the tiling thresholds from Hajnal–Szemerédi-type problems (so-named after a major result of Hajnal and Szemerédi [3] from the 1970s), neither of which are satisfactorily understood as of yet — see the surveys of Keevash [4] and Zhao [6] respectively.

Much of the attention has focussed on the 3-uniform ($r = 3$) case, with a scattering of ad hoc results but little yet in terms of an overarching theory. Goals for an internship project would include considering the case of general $r \geq 3$, constructing new examples, and giving general bounds and plausible conjectures for the asymptotic behaviour of the covering thresholds.

References

[1] Victor Falgas-Ravry, Klas Markström, and Yi Zhao. Triangle-degrees in graphs and tetrahedron coverings in 3-graphs. *Combinatorics, Probability and Computing*, 30(2):175–199, 2021.

[2] Victor Falgas-Ravry and Yi Zhao. Codegree thresholds for covering 3-uniform hypergraphs. *SIAM Journal on Discrete Mathematics*, 30(4):1899–1917, 2016.

[3] András Hajnal and Endre Szemerédi. Proof of a conjecture of P. Erdős, in âĂIJcombinatorial theory and its applicationsâĂİ(P. Erdős, A. Rényi, V. Sós, eds), 601-623, 1970.

[4] Peter Keevash. Hypergraph Turán problems. *Surveys in combinatorics*, 392:83–140, 2011.

[5] Paul Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48:436–452, 1941.

[6] Yi Zhao. Recent advances on Dirac-type problems for hypergraphs. Pp 145–165 in, *Recent trends in combinatorics*, Springer, 2016.

Gerold Jäger

Gerold Jäger is an Associate Professor of Discrete Mathematics. His research areas are discrete mathematics, combinatorial optimisation, graph theory and algorithmics. It includes the Travelling problem, game theory (especially Mastermind), theory of Latin squares and sensitivity analysis.

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Project 1: Travelling salesman problem

This is the problem of finding a shortest route visiting a given set of cities exactly once, called a tour. It is one of the most famous problems in combinatorial optimisation with many applications in transport and logistics. In this project, we develop the theoretical basis and implement heuristical algorithms, i.e., algorithms that find a short, but not necessarily the shortest tour. The used methods lie in stability analysis, i.e., how stable connections are in related problems.

Project 2: Mastermind

Mastermind is a popular board game with two players, the codemaker and the codebreaker. In the original version, the codemaker provides a code consisting of four pegs with six colours for each peg. The codebreaker has to make guesses, and each of them gets an answer from the codemaker on how close (s)he is to the correct code. Then, after a given number of guesses, the codebreaker has to provide the correct code. This project is about investigating and analysing strategies of the codebreaker to minimise the number of guesses. We consider different variants of Mastermind, e.g., when the codebreaker has to give all guesses at once (Static Mastermind) or when the code maker can give partially wrong information (Mastermind with a lie).

History and Philosophy of Mathematics

Lars-Daniel Öhman

Lars-Daniel Öhman is an Associate Professor in Mathematics. His research interests are design theory, Latin squares, graph theory, the history and philosophy of mathematics, as well as mathematics education.

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Project 1: Different notions of 'equivalence'

The concept of *equivalence* lies at the heart of mathematics. It is essential for thinking about such diverse topics as solving equations (where the two sides are "equivalent"), axiomatic foundations, and the teaching and learning of the subject. For example, two axioms of an axiomatic system are equivalent, if they give the same game when switched out for one another, while keeping all the other axioms unchanged. This type of equivalence, between axioms, is (at least at first glance) a strictly formalized type of equivalence, and as such, a part of mathematics itself.

Somewhat surprisingly, a large number of otherwise reliable sources (including, for example, the Handbook of Mathematical Induction) give incorrect accounts of the axiomatic foundations of something so simple as basic arithmetic with positive integers. In previous work, I investigated the nature of this incorrectness, at the heart of which lies the concept of equivalence of axioms. The concepts involved, I imagined, should be standard to any research level mathematician, but I learned when trying to spread the word about these incorrect accounts, that in practice, this is not as easy as it may seem. A common type of reply confused equivalence between *axioms* with equivalence between *theorems*.

This other type of equivalence, between theorems, is a much less formal concept of equivalence than that of equivalence between axioms, but still in common use in everyday talk among mathematicians. Roughly, it seems that the usage of the equivalence terminology in this case gives that two theorems are “equivalent”, if they can each be “easily derived from one another”. This concept is clearly not easily formalizable, and the confusion presented above indicates that there are very interesting and consequential questions to be investigated in this field.

Despite all the talk of “equivalence” in various forms in the mathematical literature, there has seemingly yet been no treatment of the concept from an abstract point of view, spanning the whole range of the usage.

The goal of this project is to shed further light on mathematical practice regarding the treatment of ‘equivalence’ of different types of entities in mathematical writing and reasoning. In particular, a typology of notions of equivalence will be produced, and a set of typical examples will be collected. This will provide conceptual tools of value not only for philosophers of mathematics, but also for educators, and mathematicians.

Project 2: Historical and philosophical aspects of inclusive and exclusive definitions in mathematics

In the oldest editions of Euclid’s elements, the basic geometric objects are defined such that a square is in fact not a rectangle, an ‘exclusive’ definition, whereas in almost all modern sources, squares and rectangles are included in an ‘inclusive’ hierarchy of objects. This particular shift is an eminent example of a more general historical trend, both in mathematics and in the natural sciences, where, for example, ‘humans’ have transitioned from not being included in the more general category of ‘mammals’, to being situated in the inclusive modern biological hierarchy.

The goal of this project is to more closely trace the transition from exclusive to inclusive definitions, by a study of historical sources, and by an analytical approach investigate the transition as a fundamental shift in scientific thinking. This will contribute to a key process in the history of ideas.

Mathematical Modelling and Analysis

Åke Brännström

Åke Brännström is a Professor of Interdisciplinary Mathematics. Most of his work concerns ecological and evolutionary dynamics, and typically adressed questions related to biodiversity and cooperation. He is also interested in mathematical finance.

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Project 1: Exploring multiplicative payoff functions in the evolution of cooperation

Traditionally, the evolution of cooperation is understood in terms of two distinct strategies: cooperation and defection. Recent advancements, however, recognize this as a spectrum, with full cooperation and defection at opposite ends. Within this spectrum, the possibility of evolution towards purely cooperative or defective behaviors has also been identified.

Most of this research presumes a uniformly mixed population, meaning every individual has an equal chance of encountering any other. Yet, Gylling and Brännström's 2018 study proposed a nuanced approach. They factored in the likelihood of an individual encountering others with similar strategies. Their focus was on additive payoff functions, represented by $B + C$, where B symbolizes the benefit and C denotes the cost.

Building on this foundation, our project aims to delve deeper by examining multiplicative payoff functions, characterized as $B * C$. Past research, particularly by Brännström and colleagues in 2005 and 2011, hints at an intriguing prospect: in certain environments, the tendency to associate with like-minded peers (assortment) might stimulate strategy diversification. This phenomenon has been conspicuously absent in the context of additive payoff functions.

For this project, we will engage in mathematical analyses, implement, and explore individual-based models.

Project 2: Analyzing risk and return of Swedish investment vehicles

Since 2012, Sweden's private investors have been presented with a choice for their stock market investments: the traditional custody account (AD) or the investment savings account (ISK). Each comes with its distinct tax implications:

AD: Investors are taxed 30% on all profits realized within a year, but can potentially recover between 21-30% of their realized losses.

ISK: Investors pay a fixed tax rate derived from the government interest rate, irrespective of whether their assets appreciate or depreciate in value.

At a glance, the ISK might appear to offer superior returns, but it's accompanied by heightened risk. In this project, we aim to dissect these two investment vehicles based on risk and anticipated return. Our goal is to leverage various metrics for risk-adjusted return to discern the situations in which one account might be more advantageous than the other.

Additionally, the AD enables investors to adopt responsive strategies, through which assets can be turned over by selling and buying when market prices fall below the assets' initial purchase values. This approach allows investors to recover part of the realized loss, which can then be channeled into new investments.

This project is supervised by Prof. Åke Brännström, Professor of Interdisciplinary Mathematics, and Prof. Christian Ewald, Professor of Financial Mathematics, both at Umeå University.

Antti Perälä

Antti Perälä is an Associate Professor in Mathematics. His research focuses on functional analysis, complex variables and operator theory. In particular, he is interested in various concrete operators (Toeplitz, Hankel, Volterra, etc.) on spaces of analytic functions. Common topics are their operator-theoretic properties and their spectra, as well as the related optimisation problems. He is also interested in applications to other areas of mathematics and science.

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Project 1: Hardy, Bergman and related spaces

The study of various spaces of analytic functions is a topic active ongoing research. The most common spaces are different variants of Hardy, Bergman, BMOA and Bloch spaces, as well as the Bargmann-Fock spaces of entire functions. The basic theory of all of these provides good topics for internship projects that can be adapted to the student's interests and ambitions. For instance, basic properties of weighted Bergman spaces would be a good topic blending both complex and functional analysis. The structure of these spaces ensures the existence of the Bergman kernel function, whose properties are central for many problems related to operator theory and optimisation. In many cases, this function is very difficult to calculate in closed form. Therefore, a good project could be to investigate what is known and how the lack of exact information on the Bergman kernel can be overcome.

Project 2: Carleson measures

Carleson measures are named after the Swedish mathematician Lennart Carleson, who used them in his solution to the Corona Problem. A non-negative Borel measure μ is called a p-Carleson measure for an analytic function space X, if X can be continuously embedded into L^p_μ. Characterisation of Carleson measures is a central problem in the study of analytic function spaces, and the solution is known for many different cases. A good internship project would be to study what is known and whether there are some interesting open questions.

Carleson measures have many connections to concrete operators, such as Volterra, Hankel, Toeplitz and composition operators. These topics could very well be incorporated into the project if the student so wishes. The same goes for studying the Corona Problem, or some of its variants.

Project 3: Topological vector spaces

A vector space V with a topology τ is called a topological vector space (TVS) if the vector summation and multiplication by a scalar are continuous with respect to this topology. Particularly important examples of TVS are locally convex vector spaces, which can be seen as generalisations of normed vector spaces. Many of the classical theorems for infinite dimensional Banach spaces, such as the Hahn-Banach Theorem, Uniform Boundedness Principle, and the Open Mapping Theorem, hold true for locally convex vector spaces. Investigating their proofs and other properties of these spaces would be a good internship project.

Many properties of TVS depend on the Axiom of Choice and Zorn's lemma. Another project could be studying TVS from this perspective. A good topic could be to investigate filters and ultrafilters and their connection to the topology in this setting.

Mathematical Statistics

Per Arnqvist

Per Arnqvist is an Associate Professor of Mathematical Statistics. His research areas are functional data analysis and spatial statistics. His research focuses on developing a methodology for model-based clusterization, which includes modelling, prediction and nonparametric inference, with application areas within climate reconstruction and ecology.

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Project 1: Understanding simulated ecosystem responses to management via clustering

Co-supervisor: Sarah Vollert

Sarah Vollert is a postdoctoral research fellow in Mathematical Ecology at the Integrated Science Lab, Department of Ecology, Environment and Geoscience. Her research uses differential equation models and Bayesian statistics to support environmental decision making under high uncertainty.

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Models of ecosystems are increasingly being used to support conservation and environmental management decisions by simulating millions of possible ecosystem dynamics [1]. But, communicating and interpreting large ensembles of possibilities is challenging, and summarising these with simple statistics (like median trajectory or probability of species decline; see e.g., [2]) can hide critical information about how an ecosystem might respond to management. A 30% change of "decline" could mean a small dip in populations or a full ecosystem collapse, yet these can look the same when summarised.

This project will apply clustering techniques to real ecological case studies – such as predicting the risks of introducing a predatory bird to a Pacific atoll [3]. These clustered ecosystem responses could provide more intuitive and decision-relevant descriptions of ecosystem responses to management, and could potentially help attribute responses to underlying ecological mechanisms [4], such as trophic cascades. This clustering aims to help ecologiststo better understand what their models are telling them.

References

[1] Geary, William L and Bode, Michael and Doherty, Tim S and Fulton, Elizabeth A and Nimmo, Dale G and Tulloch, Ayesha IT and Tulloch, Vivitskaia JD and Ritchie, Euan G. A guide to ecosystem models and their environmental applications. *Nature Ecology & Evolution*, 4(11):1459–1471, 2020.

[2] Baker, Christopher M and Gordon, Ascelin and Bode, Michael. Ensemble ecosystem modeling for predicting ecosystem response to predator reintroduction. *Conservation Biology*, 31(2):376–384, 2017.

[3] Vernet, Maude and Trask, Amanda E and Andrews, Caitlin E and Ewen, John G and Medina, Suzanne and Moehrenschlager, Axel and Canessa, Stefano and others. Assessing invasion risks using EICAT-based expert elicitation: application to a conservation translocation. *Biological Invasions*, 26(8):2707–2721, 2024.

[4] Lindenmayer, David B and Wood, Jeff and MacGregor, Christopher and Foster, Claire and Scheele, Ben and Tulloch, Ayesha and Barton, Philip and Banks, Sam and Robinson, Natasha and Dexter, Nick and others. Conservation conundrums and the challenges of managing unexplained declines of multiple species. *Biological Conservation*, 221:279–292, 2018.

Mehdi Moradi

Mehdi Moradi is an Associate Professor of Mathematical Statistics. He specializes in spatial and spatio-temporal point processes, change-point detection, and trajectory analysis. His research focuses on developing advanced mathematical models, that utilize change-point detection, to gain insights into the dynamics of point processes and trajectories.

Webpage: https://www.umu.se/en/staff/mehdi-moradi/

Project 1: Adaptive self-excitation models for point processes

Self-exciting point process models typically assume that the excitation mechanism is constant over the observation domain. This project aims to develop adaptive self-excitation models, allowing the excitation structure to vary across space and/or time. Such flexibility may provide a more realistic representation of heterogeneous triggering behaviour that cannot be captured by a single global excitation mechanism. The project will investigate different forms of adaptivity and their impact on model interpretation, inference, and prediction.

Project 2: Mixture models for point processes with functional marks

Point processes with functional marks arise when each event in a point pattern is associated with a function or curve. In many applications, the observed events may represent different latent groups, characterised by distinct spatial patterns and functional marks. This project aims to develop mixture models for point processes with functional marks, jointly accounting for the point locations and the associated functions. The framework will be used to identify latent components and investigate differences in their spatial and functional characteristics.

Project 3: Zero-inflated point process models on networks

Point processes on linear networks are used to model events occurring along structures such as roads, rivers, or transport systems. In many applications, however, substantial parts of the network may contain no events, potentially producing more zero-event regions than standard models would predict. This project aims to investigate zero-inflated point process models on networks, distinguishing between different mechanisms that may lead to the absence or presence of events. The goal is to develop flexible models that better represent heterogeneous event occurrence across network structures.

Project 4: Spatio-temporal modelling of parasitic infections in animals

Parasitic infections in animals provide an interesting setting where infection patterns may vary across both space and time. Standard efficacy analyses typically focus on treatment effects and repeated observations, while spatial and spatio-temporal dependence may receive limited attention. This project will investigate alternative spatio-temporal modelling approaches for analysing parasite counts and their evolution over time. Different ways of representing spatial dependence, temporal dynamics, and their interaction will be explored. The aim is to assess whether incorporating such structures provides additional insight into treatment efficacy and the dynamics of parasitic infections.

Natalya Pya Arnqvist

Natalya Pya Arnqvist is an Associate Professor of Mathematical Statistics. She has two main research interests: statistical regression modelling and functional data analysis. She is specifically interested in developing methods for shape preserving smoothing within generalized additive models and applications of shape constrained additive models (SCAMs).

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Project 1: Shape constrained generalized additive models for complex datasets

Whether we're examining how changes in water quality affect fish populations, estimating the lifespan of electronic gadgets, or exploring the link between exercise and heart health, we often rely on statistical modelling to provide us with valuable insights. When trying to analyze how a response variable connects with various predictors using flexible semi-parametric regression models, we might encounter a challenge. Sometimes, these models can become overly flexible, producing results that seem unrealistic. It is often logical to believe that some relationships follow certain shape constraints, such as monotonicity or convexity. The project aims to develop methods within the shape-constrained generalized additive modelling framework that can incorporate large datasets and a wider range of more complex relationships.

Project 2: Functional clustering methods for climate reconstruction

The growing concern regarding global warming and its associated impact on future climate patterns has heightened the need to comprehend historical climate variations over the last centuries and millennia. Understanding past climate variability is crucial for accurate climate modelling and predictions. Yet, there remains a significant knowledge gap, especially concerning changes in seasonal climate, particularly during winters, which recent climate change has disproportionately affected. Varved (annually laminated) lake sediments possess the potential to provide vital insights into past seasonal climates, thanks to their inherent annual time resolution and within-year seasonal patterns. However, the limited availability of statistical tools for quantifying and comparing these seasonal patterns has hindered their full utilisation. This project aims to develop statistical functional clustering methods, particularly tailored to the challenging complexities of varved lake sediment. The implementation of these novel functional clustering techniques, both nonparametric and parametric, will be executed within the R software environment. The project will apply the functional clustering methods to varved sediment data from lakes in Northern Sweden and Finland to infer seasonal climate changes over the last millennia.

Project 3: Advancing cosmic ray analysis: an open-source android neutron detector module

Our planet is continuously immersed in a dynamic flux of cosmic rays, spanning an extensive energy range from 10^7 eV/nucleon to 10^{20} eV/nucleon. Furthermore, the cosmic ray flux exhibits a remarkable variability, fluctuating from merely one particle per square meter and second to a solitary particle per square kilometre and century. The cosmic ray flux at Earth's orbital region is influenced by solar activity, particularly noticeable at energy levels below several tens of GeV/nucleon. This phenomenon positions temporal fluctuations in cosmic ray flux as invaluable indicators of solar activity. Ground-level instruments, such as neutron monitors, are pivotal in observing these fluctuations. Neutron monitors primarily capture secondary cosmic rays, predominantly neutrons, generated when cosmic rays interact with the Earth's atmosphere. This unique interplay between cosmic rays, solar activity, and Earth's environment has urged international collaborations to create global detector networks. The Neutron Monitor Database (NMDB) is a great demonstration of these efforts, with a key objective being global coverage. Achieving this goal entails the establishment of new monitoring stations to bridge the existing gaps. However, installing these stations can present some challenges, making cost-effective alternatives an attractive target. In light of these considerations, this project aims to conceive and rigorously evaluate an open-source Android application module designed to detect neutrons and monitor count rates from a compact neutron monitoring device. The aim is to perform a comprehensive comparative analysis using several statistical approaches, including penalized regression techniques. This analysis will also compare the performance of the proposed neutron detector with some of the existing NMDB neutron monitors.

Patrik Rydén

Patrik Rydén is a Professor of Mathematical Statistics. His research focuses on developing and applying advanced methods for analysing large and complex datasets, with applications in health sciences, life sciences, and the manufacturing industry. His methodological expertise includes supervised and unsupervised machine learning, robust optimisation, and digital twin technologies.

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Project 1: Predicting surgery times using machine learning

In collaboration with the University Hospital of Northern Sweden, this project aims to develop data-driven decision support tools for surgery, post-surgery, and subsequent care pathways. The ultimate goal is to optimise patient throughput and improve the quality of care. The project leverages machine learning to predict surgery durations using preoperative data such as medical records, prescriptions, diagnoses, clinical measures, and planned surgical procedures. These predictive tools are intended to support hospital staff in planning and resource allocation, thereby enhancing both efficiency and patient outcomes.

Project 2: Predicting ambulance driving times using machine learning

The demand for ambulance services is increasing as the number of emergency calls rises with an ageing population. In collaboration with the ambulance service in Northern Sweden, we are developing a digital twin of prehospital care to optimise resource allocation—for example, by adapting staff schedules and strategically locating ambulance stations. The project focuses on applying machine learning techniques to predict ambulance driving times using a range of data sources, including alarm location, ambulance starting point, time of day, weather conditions, predicted routes, and alarm priority. These predictive models will be integrated into the digital twin, enabling data-driven decision support to improve response times and overall efficiency in emergency care.

Project 3: Deriving p-values for predictive models

Supervised classification of large and complex datasets is widely used in many applications. Typically, data are divided into training and test sets, producing a predictive model along with an estimated performance score, such as classification accuracy. We define a model's p-value as the probability of achieving at least the observed performance by chance. The aim of this project is to develop a general approach for estimating such p-values that can be applied across a wide range of classifiers and performance metrics, including black-box algorithms. This work seeks to provide a statistically rigorous framework for assessing the significance of predictive models, thereby strengthening their reliability and interpretability in practical applications.

Sara Sjöstedt de Luna

Sara Sjöstedt de Luna is a Professor of Mathematical Statistics. Her research focuses on functional data analysis, spatial statistics and statistical methods to analyse censored and incomplete data, with applications in climatology, forestry and human movement analysis.

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Project 1: Methods to assess tree fibre length distributions in standing trees using non-destructive techniques

One of the main concerns of silviculture and forest management focuses on finding fast, cost-efficient and non-destructive ways of measuring wood properties in standing trees. Fiber length is an important trait targeted for genetic and silvicultural improvement. Such studies require large-scale non-destructive sampling, and accurate length determination. The standard procedure for non-destructive sampling is to collect increment cores (cylindric wood samples of diameter 5mm), singularize their cells by maceration, and then measure them. The data can be measured by an optical fibre analyser (OFA) or by microscopy. Increment core data analysed by OFAs consists of the cell lengths of both cut and uncut fibres (tracheids) and fines (such as ray parenchyma cells) without being able to identify which cells are cut or if they are fines or fibres. The microscopy measured data consist of the observed lengths of the uncut fibres in the increment core. Since fibre lengths may be longer than 5 mm there is a selection bias to tackle as well, both for the OFA and the microscopy data.

We have addressed the problem of reconstructing the true fibre length distributions in the standing tree from these types of data in some articles, see below. We proposed a censored version of a mixture of the fine and fibre length distributions to fit the OFA data, and selection bias adjusted fibre length distributions to fit the microscopy data, under distributional assumptions. Two choices for the assumptions of the underlying density functions have so far been considered, the generalized gamma and the lognormal densities. Maximum likelihood estimation and an EM algorithm was used to estimate the model parameters for the OFA data and the microscopy data. The methods are implemented in the R package fiberLD.

It is of interest to allow a larger variety of underlying distributions than the generalized gamma and lognormal distributions. We propose an internship project that extends and develops the methods for additional underlying distributions and implements them in the fiberLD R package.

Jun Yu

Jun Yu is a full professor of Mathematical Statistics with focus on Statistical Learning and Inference for Spatiotemporal Data, including fundamental research and applications in AI.

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Project 1: Interpretable Machine Learning - Generative mixture of linear models by DNN co-supervision

Co-supervisor: Xin Huang

Artificial intelligence (AI) has become ubiquitous in our daily lives, even though we are often not aware of the technology at work. Machine learning (ML) is the technical core of AI. With the enormous success of deep neural networks (DNNs), the research focus of ML has shifted from pursuing high accuracy to a few other qualities of an ML system. One of the highly valued traits of an ML system is interpretability. For some critical tasks, a black-box ML classifier is rejected even if it performs the best on a test dataset. The caution against black-box ML is well grounded since a test dataset usually cannot fully represent the phenomenon under study. As a proof of concept, we have developed a prototype that approximates a DNN model's predictions with a piecewise-linear function (i.e., linear decision boundaries), called the Mixture of Linear Models (MLM).

This project aims to further develop MLM by incorporating more sophisticated region-specific models and enhancing module training within MLM. These improvements will expand the method's potential applications and increase prediction accuracy. This project will contribute novel algorithms for interpretable ML and create a platform for a formal study of the relationship between interpretability and prediction accuracy.

Some prior knowledge in the Fourier transform, wavelet transform, spatiotemporal statistics, generalised additive models, and neural networks is beneficial. So are programming skills in Python and R.

Project 2: Intelligent data sampling for phytoplankton monitoring

Marine ecosystems provide numerous ecosystem services that sustain the human economy but are threatened by anthropogenic impacts (climate change, eutrophication). Monitoring programs address and measure changes in these systems that steer ecological management. Phytoplankton constitute the base of aquatic ecosystems, and their importance is such that European directives govern their environmental status. Phytoplankton can experience short-term changes (on an hourly scale). In contrast, most monitoring programs currently have fortnightly or lower resolution, making them insufficient to describe dynamic aspects crucial to the ecosystem. By applying sparse statistical learning and intelligent data sampling techniques, this project aims to determine the optimal sampling frequency for detecting externally driven phytoplanktonic change while also providing autonomous solutions to optimise the cost of monitoring programmes. These approaches will contribute to a comprehensive framework for improving the design and interpretation of monitoring programs.

Some prior knowledge in the Fourier transform, wavelet transform, statistical signal analysis, and sparse statistical modelling is beneficial. So are programming skills in Python and R.

This is a collaborative project with Dr Léon Serre-Fredj at Umeå Marine Sciences Centre (UMSC) https://www.umu.se/en/staff/leon-serre-fredj/.

Project 3: Bayesian hierarchical model for blind source separation of biomedical signals

Your brain plans, organises, and sends commands to your muscles via the spinal cord and the motor neurons. As you increase your force, more motor neurons will be activated, and their activity will increase individually. This activity can be measured from the skin using electrodes by decomposing the electrical activity using so-called blind source separation algorithms. This information can be used to study motor control, neuromuscular diagnosis, and rehabilitation. The state-of-the-art blind source separation algorithms in this field are data-driven and exploit the sparsity of the underlying motor neuron activity. However, current methods do not exploit the rich a priori information about these underlying sources and require manual post-processing to overcome the non-quantifiable estimation uncertainty. This project aims to develop Bayesian hierarchical models to tackle uncertainty estimation problems. The developed models will be compared against state-of-the-art methods and evaluated using sophisticated simulation models and experimental data with available ground truth.

Some prior experience with statistical signal analysis, spatiotemporal statistics, and Bayesian hierarchical models is beneficial. So are programming skills in Python, R, or MATLAB.

This is a collaborative project with Dr. Robin Rohlén at the Department of Diagnostics and Intervention: https://www.umu.se/en/staff/robin-rohlen/

Project 4: Statistical Learning for Chronosilviculture

Time, and in particular 'right' timing of (their) important life processes, matters to all organisms, and most possess a circa 24 h (circadian) clock, a mechanism that enables them to coordinate their internal physiology with temporal changes in environmental light and temperature. For plants, this is critical to their strategic decisions about growth and development across both daily and seasonal time frames. To cultivate healthy plants and optimise harvest yields, knowledge of plant circadian and seasonal growth can be applied in the emerging field of 'Chronoculture'. This project aims to develop statistical learning approaches to understand the role of the circadian clock, from gene expression to growth, with a focus on the explainability and interpretability of our models.

Some prior knowledge in generalised additive models, time series analysis, and high-dimensional statistics is beneficial. So are programming skills in Python and R.

This is a collaborative project with Dr Maria E Eriksson at the Department of Plant Physiology: https://www.umu.se/en/staff/maria-eriksson/

Project 5: AI-powered computational materials design enabling efficient development and implementation of sustainable metals

The development and implementation of industrial materials today are, to a large extent, tedious trial-and-error processes. Furthermore, in many materials industries, e.g., the metal industry, it is difficult to replicate full-scale manufacturing at the pilot or laboratory scale. This means that extensive, large-scale trial-and-error development/manufacturing is needed to bring novel metal products into use today. This is clearly unsustainable, leading to the consumption of large quantities of energy and raw materials and causing significant emissions even before the materials are produced. Furthermore, we urgently need new sustainable metal products produced using fossil-free manufacturing routes, where critical raw materials have been substituted, a higher fraction of recycled raw material is used, and also where different performance aspects are increased, for example, in transportation, novel high-strength materials are needed with a higher strength-to-weight ratio to enhance transport capability. Therefore, we need to develop further effective methods for the development and implementation of sustainable metals to address these challenges. This project aims to develop novel approaches for computational materials design using statistical learning with sparsity and advance our understanding of material phenomena that are often complex-structured and high-dimensional.

Some prior knowledge in optimal experimental designs and sparse statistical modelling is beneficial. So are programming skills in Python and R.

This is a collaborative project with Prof Peter Hedström at the Department of Materials Science and Engineering at KTH: https://www.kth.se/profile/pheds

Mathematical Foundations of Artificial Intelligence

Axel Kisch Flinth

Axel Kisch Flinth is an Assistant Professor in Mathematics. His research concerns mathematical aspects of signal processing and machine learning.

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Project 1: Security of bilinear communication

In recent work, my collaborators and I have proposed and analysed a novel technique for secure communications based on the solution of *sparse bilinear inverse problems.* The idea is to 'hide' a linear encryption of a message through *convolving* it with a random filter. With the correct design of filters, the legitimate users can still recover the message, while the convolution should, at least intuitively, make a malicious 'man-in-the-middle-attack' harder.

We have thus far been able to show some theoretical results about the hardness of recovering the linear encoding operator, in particular in the idealised case of allowing the attacker to study the channel for a long time. We have also designed a practically viable technique, based on covariance estimation, to practically attack the protocol.

In this project, the aim is to derive and study new classes of such practical algorithms. Depending on the intern's interests and experience, one can consider different types of attacks, as well as put a higher emphasis on either empirical experiments or theoretical investigation. An interesting direction is to try to train machine learning models to break the code.

Project 2: Identifiability of neural ODEs

Neural networks are the workhorse of the recent AI revolution, but the mathematical theory about them is still under construction. One important open question is that of *identifiability*: Given the end-to-end function of a neural network, can its parameters be recovered? This question relates to symmetries in the parameter space of neural networks, which in turn is related to their behavior during training.

Very recently, a lot of attention has shifted to so-called *residual networks*. It seems that their special structure make mathematical analysis of them more tractable, in particular for the identifiability question. The purpose of this project is therefore to study identifiability of their *infinitely deep counterparts*, the *neural ODEs*. The idea of a neural ODE is to train a vector field v_t on the data domain, and then use the flow defined by it to define an end-to-end map:

$$\begin{cases} \dot{x} & = v_t(x) \\ x(0) & = x_0 \end{cases}, \quad f_v(x_0) = x(1).$$

The question of identifiability in this setting is thus: Given an end-to-end map f_v, is it possible to recover v? It is easy to show that if the vector fields are completely unrestricted, one cannot expect this. The intern will thus be tasked with investigating this problem when the vector fields are restricted to certain classes – such as polynomials, time-independent, and so on. This project will be conducted in collaboration with Fredrik Ohlsson, an Associate Professor in Mathematics, and a specialist in neural ODEs, https://www.umu.se/en/staff/fredrik-ohlsson/.

Alp Yurtsever

Alp Yurtsever is an Associate Professor in Optimization and Machine Learning. His research focuses on developing theoretical foundations and algorithms for large-scale optimization problems with applications in machine learning and data science. His work addresses challenges such as vast datasets potentially distributed across extensive networks with restricted access, high-dimensional parameter spaces, and complex problem models involving non-smooth and non-convex loss functions.

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Project 1: Privacy aspects of Frank-Wolfe-type updates in federated learning

The demand for privacy-preserving approaches in Machine Learning (ML) has become increasingly critical in recent years. Federated Learning (FL) has emerged as a distributed ML paradigm designed to address these privacy concerns while enabling collaborative model training across a large network of clients [1]. By shifting the computational burden to edge devices, FL allows models to be trained locally on private data, making it a vital framework for solving large-scale optimization and learning problems in a distributed manner while upholding privacy principles.

However, FL is associated with significant computational and communication costs. One of the primary objectives in the field is to design optimization algorithms capable of efficiently handling a large number of parameters, particularly in complex learning tasks where parameters are subject to constraints.

In our recent work [2], we introduced a projection-free algorithm called FedFW for constrained FL problems. The FedFW algorithm aggregates Frank-Wolfe (FW) step directions at the server, providing low per-iteration computational cost and facilitating the communication of sparse signals. Preliminary results on the privacy of communicating FW steps instead of gradients are reported in [2], demonstrating the superiority of sharing FW steps over gradient inversion attacks—such as the Deep Leakage algorithm [3]—for the CIFAR100 dataset.

This internship project will focus on a deeper investigation of the privacy aspects of sharing FW steps in an FL setting. The intern will theoretically characterise the privacy benefits of FW-based updates and extend the existing empirical analysis to evaluate their robustness under different adversarial models.

References

[1] Jakub Konečný, H Brendan McMahan, Daniel Ramage, and Peter Richtárik. Federated optimization: Distributed machine learning for on-device intelligence. *arXiv preprint arXiv:1610.02527*, 2016.

[2] Ali Dadras, Sourasekhar Banerjee, Karthik Prakhya, and Alp Yurtsever. Federated Frank-Wolfe algorithm. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, pages 58–75. Springer, 2024.

[3] Ligeng Zhu, Zhijian Liu, and Song Han. Deep leakage from gradients. *Advances in neural information processing systems*, 32, 2019.

Project 2: Convex reformulations via copositive programming

Copositive programming is an important framework in convex optimization as it allows reformulating many non-convex optimization problems of interest as convex optimization problems in an exact sense [1]. However, despite this convexity, these problems remain intractable due to the complexity of the underlying cone constraint [2]. Recent advancements in quantum annealers offer promising directions to address this intractability by providing new tools for solving certain classes of optimization problems on this cone efficiently [3].

This internship project will explore convex reformulations of selected non-convex optimization problems through copositive programming and evaluate the efficiency of existing solvers. The intern will also investigate alternative relaxations of these formulations that can be efficiently solved using classical computing methods, comparing their performance with quantum-based approaches.

References

[1] Mirjam Dür. Copositive programming–a survey. In *Recent Advances in Optimization and its Applications in Engineering: The 14th Belgian-French-German Conference on Optimization*, pages 3–20. Springer, 2010.

[2] Immanuel M Bomze, Mirjam Dür, Etienne De Klerk, Cornelis Roos, Arie J Quist, and Tamás Terlaky. On copositive programming and standard quadratic optimization problems. *Journal of Global Optimization*, 18:301–320, 2000.

[3] Alp Yurtsever, Tolga Birdal, and Vladislav Golyanik. Q-fw: A hybrid classical-quantum frank-wolfe for quadratic binary optimization. In *European Conference on Computer Vision*, pages 352–369. Springer, 2022.