Internship Programme

Project list
Autumn term 2024/25

Department of Mathematics and Mathematical Statistics
Umeå University, Umeå, Sweden

About the programme

Our program is a one-term adventure based in Umeå, where you will be working directly with researchers in your selected field. This comprehensive program is an opportunity to bridge the gap between academic education and its application in the research environment.

Eligibility and Applications

The program is primarily directed to graduate students from our partner universities, but students from all universities within the Erasmus+ network are also eligible to apply.

Spring application round: March 1st – March 30th

The student applies for **one** specific project that is best aligned with student's educational background and research interest. Interested students should submit their applications via online form. Together with online application the student needs to submit the following documents:

- CV
- Transcript of record (in English)
- Motivation letter

Support Provided

- Successful applicants have their round-trip fare (economy class) to Umeå covered by our department.
- We help the intern secure student accommodation (the intern covers the accommodation costs).
- We assist interns in applying for the Erasmus+ stipend from their home institution.
- When in Umeå, our interns are provided with a conducive work environment, including a workspace, necessary working materials, and insurance.

Selection Procedure

Applications are reviewed by a dedicated working group which, together with prospective supervisors, chooses the most eligible candidates based on their eligibility, academic record and compatibility with the selected project.

Content of the Document

This document contains information about available projects in the specific round (and corresponding supervisors), and the applicants must choose one specific project. The projects are organised by research topics and supervisors.

For further details and application form the programme, please see:

https://www.umu.se/en/department-of-mathematics-and-mathematical-statistics/education/internship-programme/

Discrete Mathematics

Victor Falgas Ravry

Victor Falgas Ravry is an Associate Professor in Mathematics. His main research interests lie in extremal combinatorics and discrete probability — interests which are shared by several others members of the Discrete Mathematics group at Umeå. Typical topics include extremal graph theory, Ramsey theory, games on graphs and random graph models.

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Project 1: Maker–Breaker percolation games

Maker–Breaker games were introduced in an influential paper of Chvátal and Erdős [2] in the late 1970s and form a rich and widely studied class of positional games.

The board in a Maker–Breaker game consists of a set B (for example, the set of edges of a graph, or the squares in a tic-tac-toe grid). Two players, known as Maker and Breaker, take turn claiming elements of the board B. Maker's goal is to claim all elements of one member W of a pre-specified family $\mathscr{W} \subseteq \mathscr{P}(B)$ of winning sets (for example, a winning set could be all the edges of a triangle, or three squares on the same row, column or diagonal in a tic-tac-toe grid). Breaker's aim is to prevent this.

As the vague and general description above suggests, there are many games within this class, and many questions one could ask: who has a winning strategy under optimal play? how long will the game last? what happens if one of the players selects his or her moves at random? what happens if we allow one of the players to claim more elements of the board per turn than the other player?

A rich theory has developed in an attempt to answer these questions (see e.g. the book of Beck [1] and the survey of Stojaković and Szabó [6]), and to elucidate some of the intriguing connections between these games and deep phenomena in discrete probability and Ramsey theory. As Tibor Szabó has pointed out, these delightfully fun and deceptively light-hearted combinatorial games have contributed to many of the most significant advances in discrete mathematics in the past half century.

A few years ago, I introduced a new family of Maker–Breaker games with motivation coming from percolation theory. These games are played on a (large) graph G with two specified vertices u and v. Maker and Breaker take turns claiming (some number of) edges of G, and Maker's aim is to build a path joining u to v.

These Maker–Breaker percolation games may be viewed as generalisations of the classical Shannon switching game, and have been the subject of two articles by my postdoc Nicholas Day and myself [4, 3], and more recently of a lovely paper of Dvořák, Mond and Souza [5]. As can be seen from the concluding sections of these three papers, a great many questions remain open — many of which would be highly suitable for an internship, including in particular some as-yet unstudied site percolation versions (where Maker and Breaker claim vertices rather than edges of G).

References

[1] József Beck. *Combinatorial games: tic-tac-toe theory*, volume 114. Cambridge University Press Cambridge, 2008.

[2] Vašek Chvátal and Paul Erdős. Biased positional games. In *Annals of Discrete Mathematics*, volume 2, pages 221–229. Elsevier, 1978.

[3] A. Nicholas Day and Victor Falgas-Ravry. Maker–breaker percolation games I: crossing grids. *Combinatorics, Probability and Computing*, 30(2):200–227, 2021.

[4] A. Nicholas Day and Victor Falgas-Ravry. Maker-breaker percolation games II: Escaping to infinity. *Journal of Combinatorial Theory, Series B*, 151:482–508, 2021.

[5] Vojtěch Dvořák, Adva Mond, and Victor Souza. The maker-breaker percolation game on the square lattice. *Journal of Combinatorial Theory, Series B*, to appear 2024.

[6] Miloš Stojaković and Tibor Szabó. Positional games on random graphs. *Random Structures & Algorithms*, 26(1-2):204–223, 2005.

Project 2: Random subcube intersection graphs

Given a d-dimensional hypercube $Q_d := \{0,1\}^d$ and an integer k: $0 \leq k \leq d$, let $\mathrm{Subcube}_k(Q_d)$ denote the collection of all k-dimensional subcubes of Q_d.

Construct a graph $G_k(d)$ on $\mathrm{Subcube}_k(Q_d)$ by joining two subcubes from $\mathrm{Subcube}_k(Q_d)$ by an edge if they have a non-empty intersection. The resulting *subcube intersection graph* can be thought of as a hypercube analogue of the (complement of) well-studied Kneser graph.

With motivation coming from social choice theory, Markström and I [1] introduced certain random subcube intersection graph models related to site percolation on $G_k(d)$. A number of fundamental questions about these models remain open however, including connectivity and component evolution for site percolation on $G_k(d)$. Addressing questions not addressed in [1] would be a highly suitable internship project, and would almost certainly involve investigating the separate, unstudied and intriguing problems of deriving chromatic, isoperimetric and spectral results for $G_k(d)$ — in particular obtaining analogues of a seminal result of Lovász [4] on the Kneser graph and of a classical inequality in Q_d due to Harper [2] in the 'subcube Kneser' setting. Further possibilities include extending earlier Ramsey- and Turán-tye results of Johnson and Markström [3].

References

[1] Victor Falgas-Ravry and Klas Markström. Random subcube intersection graphs I: Cliques and covering. *The Electronic Journal of Combinatorics*, pages P3–43, 2016.

[2] Lawrence H. Harper. Optimal numberings and isoperimetric problems on graphs. *Journal of Combinatorial Theory*, 1(3):385–393, 1966.

[3] J. Robert Johnson and Klas Markström. Turán and Ramsey properties of subcube intersection graphs. *Combinatorics, Probability and Computing*, 22(1):55–70, 2013.

[4] László Lovász. Kneser's conjecture, chromatic number, and homotopy. *Journal of Combinatorial Theory, Series A*, 25(3):319–324, 1978.

Project 3: Bootstrap percolation in random intersection graphs

Bootstrap percolation is concerned with the study of a family cellular automata. It first arose in work of Chalupa and Leath and Reich [1] on magnetic phenomena, and has been widely studied as a toy model for the spread of an infection through a network.

Given a graph G, an infection threshold r and a set of initially infected vertices I_0, a bootstrap percolation process on G unfolds as follows: at each time $t \geq 0$, all vertices of G that are adjacent to at least r vertices of the infected set I_t become infected and are added to I_t to form I_{t+1}. The set of eventually infected vertices in this process is denoted by $I_\infty := \bigcup_{t\geq 0} I_t$.

There are many questions one could ask: given an initially infected set I_0 of a given size, how large/how small could the size of the eventual infection $|I_\infty|$ be? How long does the infection take to reach its final state? If the set of initially infected vertices I_0 is selected at random, what is the likely distribution of I_∞? What if the threshold for infection r is not uniform, or if the infection spreads in a random rather than a deterministic fashion?

As part of an internship project, I propose to study bootstrap percolation on random intersection graphs, to provide counterparts in that setting to the work of Janson, Łuczak, Turova and Vallier [3] on the Erdős–Rényi random graph model, and of Whittemore [4] and Sarkar and myself [2] on the Gilbert random geometric graph model.

References

[1] John Chalupa, Paul L Leath, and Gary R Reich. Bootstrap percolation on a Bethe lattice. *Journal of Physics C: Solid State Physics*, 12(1):L31, 1979.

[2] Victor Falgas-Ravry and Amites Sarkar. Bootstrap percolation in random geometric graphs. *Advances in Applied Probability*, 55(4), 1254–1300, 2023.

[3] Svante Janson, Tomasz Łuczak, Tatyana Turova, and Thomas Vallier. Bootstrap percolation on the random graph $G_{n,p}$. *The Annals of Applied Probability* 22(5), 1989–2047, 2012. 2012.

[4] Alyssa Whittemore. *Bootstrap percolation on random geometric graphs*. PhD thesis, The University of Nebraska-Lincoln, 2021.

Project 4: Mixed degree conditions for hypergraphs

An r-uniform hypergraph (or *r-graph*) is a pair $H = (V, E)$, where $V = V(H)$ is a set of vertices, and $E = E(H)$ is a collection of r-sets from V that form the edges of H. The *degree* $\deg(S)$ of a set S in H is the number of edges of H that contain S. The minimum of $\deg(S)$ over all subsets $S \subseteq V$ of size $|S| = \ell$ is known as the *minimum ℓ-degree* of H, and is denoted by $\delta_\ell(H)$.

A copy of the clique on t vertices $K_t^{(r)}$ in H is a t-set $X \subseteq V$ such that all $\binom{t}{r}$ possibles r-sets from X are present as edges in H. If H contains no such copy, H is said to be $K_t^{(r)}$*-free*.

A classical problem in extremal hypergraph theory is: given integers $1 \leq \ell < r < t$, determine the maximum value of $\delta_\ell(H)$ over all $K_t^{(r)}$-free r-graphs H on $n = |V(H)|$ vertices. The case $r = 2$ was resolved almost a century ago in a celebrated paper of Turán [5].

For all $r \geq 3$, however, this problem — known as the *hypergraph Turán problem* —- has remained stubbornly open in all cases, and is arguably one of the most significant open problems in discrete mathematics; see Keevash's survey [1] dedicated to it.

As part of an internship, I proposed to study a mixed-degree version of the hypergraph Turán problem recently considered by Markström, Sliacan and I. The focus would be on obtaining good lower bound constructions for this new family of problems, which is as yet unstudied, in order to cast light on the hypergraph Turán problem. Part of the work would build on a mountain of unpublished data on flag algebraic bounds computed by Sliacan, and on a study of the treasure trove of constructions provided by Sidorenko in his 1995 survey [2].

References

[1] Peter Keevash. Hypergraph Turán problems. *Surveys in combinatorics*, 392:83–140, 2011.

[2] Alexander Sidorenko. What we know and what we do not know about Turán numbers. *Graphs and Combinatorics*, 11(2):179–199, 1995.

[3] Paul Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48:436–452, 1941.

Project 5: Covering problems in hypergraphs

An r-uniform hypergraph (or r-*graph*) is a pair $H = (V, E)$, where $V = V(H)$ is a set of vertices, and $E = E(H)$ is a collection of r-sets from V that form the edges of H. The *degree* $\deg(S)$ of a set S in H is the number of edges of H that contain S. The minimum of $\deg(S)$ over all subsets $S \subseteq V$ of size $|S| = \ell$ is known as the *minimum ℓ-degree* of H, and is denoted by $\delta_\ell(H)$.

A copy of the clique on t vertices $K_t^{(r)}$ in H is a t-set $X \subseteq V$ such that all $\binom{t}{r}$ possibles r-sets from X are present as edges in H. A $K_t^{(r)}$-*covering* in H is a collection of copies of $K_t^{(r)}$ that together cover all the vertices of H. If these copies may in addition be chosen to be vertex-disjoint (which implies in particular that $|V(H)|$ must be divisible by t), then we say that H contains a $K_t^{(r)}$-*tiling* (also known as a $K_t^{(r)}$-*factor*).

A natural question in extremal hypergraph theory is: given $1 \leq \ell < r < t$, what is the least d such that every r-graph on n vertices with minimum ℓ-degree strictly greater than d must contain a $K_t^{(r)}$-covering? We denote the answer to this question by $c_\ell(n, K_t^{(r)})$, and refer to it as the *ℓ-degree covering threshold* for $K_t^{(r)}$.

The systematic study of covering thresholds was initiated in works of Zhao and myself [2] (in the case $r = 3$, $\ell = 2$) and Markström, Zhao and myself [1] (in the case $r = 3$, $\ell = 1$), and has attracted an increasing amount of attention in recent years. Covering thresholds lie between two of the most studied families of parameters in extremal hypergraph theory, namely the existence thresholds studied in Turán-type problems (so-named after the pioneering work of Turán [5] discussed in the previous subsection) and the tiling thresholds from Hajnal–Szemerédi-type problems (so-named after a major result of Hajnal and Szemerédi [3] from the 1970s), neither of which are satisfactorily understood as of yet — see the surveys of Keevash [4] and Zhao [6] respectively for more details.

Much of the attention has focussed on the 3-uniform ($r = 3$) case, with a scattering of ad hoc results but little as of yet in terms of an overarching theory. Goals for an internship project would include considering the case of general $r \geq 3$, constructing new examples, and giving general bounds and plausible conjectures for the asymptotic behaviour of the covering thresholds.

References

[1] Victor Falgas-Ravry, Klas Markström, and Yi Zhao. Triangle-degrees in graphs and tetrahedron coverings in 3-graphs. *Combinatorics, Probability and Computing*, 30(2):175–199, 2021.

[2] Victor Falgas-Ravry and Yi Zhao. Codegree thresholds for covering 3-uniform hypergraphs. *SIAM Journal on Discrete Mathematics*, 30(4):1899–1917, 2016.

[3] András Hajnal and Endre Szemerédi. Proof of a conjecture of P. Erdős, in *Combinatorial theory and its applications* (P. Erdős, A. Rényi, V. Sós, eds), 601-623, 1970.

[4] Peter Keevash. Hypergraph Turán problems. *Surveys in combinatorics*, 392:83–140, 2011.

[5] Paul Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48:436–452, 1941.

[6] Yi Zhao. Recent advances on Dirac-type problems for hypergraphs. Pp 145–165 in, *Recent trends in combinatorics*, Springer, 2016.

Project 6: Extremal multigraph theory

Say that a multigraph G is (s, q)*-sparse* if every s-set of vertices in G supports at most q edges, counting multiplicities. How large can the product of the edge multiplicities of G be if G is an (s, q)-sparse multigraph on n vertices?

This question was posed by Mubayi and Terry in 2016, with motivation coming from hypergraph container theory. It can also be seen as a natural attempt to generalise classical results from extremal graph theory (in particular the celebrated theorem of Turán) to the setting of multigraphs with bounded edge multiplicities. That one seeks to maximise a product results in some unusual features and challenges, including extremal constructions with parts containing an asymptotically transcendental proportion of the vertices.

Despite recent progress, the Mubayi–Terry problem remains poorly understood for general (s, q). In this project, we will seek both to complete the general picture and to address related questions on hereditary properties of graphs with bounded multiplicities.

References

[1] A.N. Day, V. Falgas–Ravry and A. Treglown, *Extremal problems for multigraphs*, Journal of Combinatorial Theory, Series B **154** (2022), 1–48.

[2] V. Dvořák, V. Falgas–Ravry, A. Mond and V. Souza, *On transitions and generalised cycles for the Mubayi–Terry problem*, preprint (2023).

[3] V. Falgas–Ravry, *On an extremal problem for locally sparse multigraphs*, to appear in European Journal of Combinatorics.

[4] Z. Füredi, and A. Kündgen, *Turán problems for integer-weighted graphs*, Journal of Graph Theory **40(4)** (2002), 195–225.

[5] D. Mubayi and C. Terry, *An extremal graph problem with a transcendental solution*, Combinatorics, Probability and Computing, **28(2)** (2021), 303–324.

[6] D. Mubayi and C. Terry, *Extremal theory of locally sparse multigraphs*, SIAM Journal on Discrete Mathematics **34(3)** (2020), 1922–1943.

Gerold Jäger

Gerold Jäger is an associate professor of discrete mathematics at the department. His research areas are discrete mathematics, combinatorial optimization, graph theory and algorithmics. It includes the Traveling problem, game theory (especially Mastermind), theory of Latin squares and sensitivity analysis.

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Project 1: Traveling Salesman Problem

This is the problem of finding a shortest route visiting a given set of cities exactly once, called a tour. It is of the most famous problems in combinatorial optimization with many applications in transport and logistics. In this project we develop the theoretical basis and implement heuristical algorithms, i.e., algorithms that find a short, but not necessarily shortest tour. The used methods lie in stability analysis, i.e., how stable connections are in related problems.

Project 2: Mastermind

Mastermind is a popular board game with two players, the codemaker and the codebreaker. In the original version, the codemaker provides a code consisting of four pegs with six colours for each peg. The codebreaker has to make guesses and each of them gets an answer from the codemaker how close (s)he is from the correct code. Then after a given number of guesses the codebreaker has to provide the correct code. This project is about investigating and analyzing strategies of the codebreaker to minimize the number of guesses. We consider different variants of Mastermind, e.g., when the codebreaker has to give all guesses at once (Static Mastermind) or when the codemake can give partially wrong infomation (Mastermind with a lie).

Mathematical Modelling and Analysis

Åke Brännström

Åke Brännström is a Professor of Interdisciplinary Mathematics. Most of his work concerns ecological and evolutionary dynamics, and typically adressed questions related to biodiversity and cooperation. He is also interested in mathematical finance.

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Project 1: Exploring multiplicative payoff functions in the evolution of cooperation

Traditionally, the evolution of cooperation is understood in terms of two distinct strategies: cooperation and defection. Recent advancements, however, recognize this as a spectrum, with full cooperation and defection at opposite ends. Within this spectrum, the possibility of evolution towards purely cooperative or defective behaviors has also been identified.

Most of this research presumes a uniformly mixed population, meaning every individual has an equal chance of encountering any other. Yet, Gylling and Brännström's 2018 study proposed a nuanced approach. They factored in the likelihood of an individual encountering others with similar strategies. Their focus was on additive payoff functions, represented by $B + C$, where B symbolizes the benefit and C denotes the cost.

Building on this foundation, our project aims to delve deeper by examining multiplicative payoff functions, characterized as $B * C$. Past research, particularly by Brännström and colleagues in 2005 and 2011, hints at an intriguing prospect: in certain environments, the tendency to associate with like-minded peers (assortment) might stimulate strategy diversification. This phenomenon has been conspicuously absent in the context of additive payoff functions.

For this project, we will engage in mathematical analyses, implement, and explore individual-based models.

Project 2: Analyzing risk and return of Swedish investment vehicles

Since 2012, Sweden's private investors have been presented with a choice for their stock market investments: the traditional custody account (AD) or the investment savings account (ISK). Each comes with its distinct tax implications:

AD: Investors are taxed 30% on all profits realized within a year, but can potentially recover between 21-30% of their realized losses.

ISK: Investors pay a fixed tax rate derived from the government interest rate, irrespective of whether their assets appreciate or depreciate in value.

At a glance, the ISK might appear to offer superior returns, but it's accompanied by heightened risk. In this project, we aim to dissect these two investment vehicles based on risk and anticipated return. Our goal is to leverage various metrics for risk-adjusted return to discern the situations in which one account might be more advantageous than the other.

Additionally, the AD enables investors to adopt responsive strategies, through which assets can be turned over by selling and buying when market prices fall below the assets' initial purchase values. This approach allows investors to recover part of the realized loss, which can then be channeled into new investments.

This project is supervised by Prof. Åke Brännström, Professor of Interdisciplinary Mathematics, and Prof. Christian Ewald, Professor of Financial Mathematics, both at Umeå University.

Christian Ewald

Christian Ewald is a Professor of Financial Mathematics with specialization in Financial Mathematics, Quantitative Finance, Risk-Management, and Commodities. He is also affiliated with the HINN Business Analytics Group (Norway) as well as the Adam Smith Business School, University of Glasgow (UK).

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Project 1: Inflation-indexed securities and Derivatives: Applications in Fund Management

Inflation-indexed securities are securities whose cash flows are linked to the level of inflation. They fulfill three key functions: First, for governments, they reduce borrowing costs as they allow to avoid a costly inflation risk premium. Second, for investment funds, in particular pension funds, they allow for inflation hedging. Third, with a good pricing model, they allow to make relative short-term forecasts of inflation levels, where macro-economic models often fail. In this project prices as well as relevant indices for inflation-indexed securities will be used to make short-term inflation forecasts for different regions. In a continuation of this project or possibly a second project the impact of short-term inflation on currency exchange rates can be investigated.

Mathematical Statistics

Mehdi Moradi

Mehdi Moradi is an Associate Professor of Mathematical Statistics. He specializes in spatial and spatio-temporal point processes, change-point detection, and trajectory analysis. His research focuses on developing advanced mathematical models, that utilize change-point detection, to gain insights into the dynamics of point processes and trajectories.

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Project 1: Change-point detection for point processes

Significant advancements in data collection and storage capacities have led to vast spatial point data availability from diverse sources, which often give access to (precise) spatial locations and time occurrences of events together with further valuable point-specific information. Applications, among other things, include e.g. locations of traffic accidents and street crimes. The nature of correlations between these events can fluctuate over time, hinting at varying forms of interaction among events during different time intervals. This project is dedicated to the identification of such changes and the classification of point patterns through second-order analysis.

Project 2: Edge correction for Voronoi-based intensity estimators

The spatial distribution of point patterns, which give locations of events such as e.g. traffic accidents and street crimes, is usually estimated via kernel- and Voronoi-based approaches. It has recently been shown that a resample-smoothed version of Voronoi estimators outperforms kernel-based estimators in terms of both bias and variance. However, that estimator suffers from border issues leading to under/over estimations near the borders of the observed area. This project works on proposing an edge-corrected version of the resample-smoothed

Voronoi estimators.

Project 3: Movement analysis with a focus on traffic data

Analyzing traffic data is crucial for optimizing transportation systems, enhancing safety, and reducing congestion. It provides valuable insights into traffic patterns, allowing authorities to make informed decisions about infrastructure improvements, traffic management strategies, and public transit enhancements. This project focuses on developing point-process-based methods to study how the underlying network (e.g., street network) affects traffic flow and human movement while also quantifying the influence of specific segments (e.g., highways) on traffic dynamics.

Natalya Pya Arnqvist

Natalya Pya Arnqvist is an Associate Professor of Mathematical Statistics. She has two main research interests: statistical regression modelling and functional data analysis. She is specifically interested in developing methods for shape preserving smoothing within generalized additive models and applications of shape constrained additive models (SCAMs).

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Project 1: Shape constrained generalized additive models for complex datasets

Whether we're examining how changes in water quality affect fish populations, estimating the lifespan of electronic gadgets, or exploring the link between exercise and heart health, we often rely on statistical modelling to provide us with valuable insights. When trying to analyze how a response variable connects with various predictors using flexible semi-parametric regression models, we might encounter a challenge. Sometimes, these models can become overly flexible, producing results that seem unrealistic. It is often logical to believe that some relationships follow certain shape constraints, such as monotonicity or convexity. The project aims to develop methods within the shape constrained generalized additive modelling framework that can incorporate large datasets and a wider range of more complex relationships.

Project 2: Functional clustering methods for climate reconstruction

The growing concern regarding global warming and its associated impact on future climate patterns has heightened the need to comprehend historical climate variations over the last centuries and millennia. Understanding past climate variability is crucial for accurate climate modelling and predictions. Yet, there remains a significant knowledge gap, especially concerning changes in seasonal climate, particularly during winters, which recent climate change has affected disproportionately. Varved (annually laminated) lake sediments possess the potential to provide vital insights into past seasonal climates, thanks to their inherent annual time resolution and within-year seasonal patterns. However, the limited availability of statistical tools for quantifying and comparing these seasonal patterns has hindered their full utilization. This project aims to develop statistical functional clustering methods, particularly tailored to the challenging complexities of varved lake sediment. The implementation of these novel functional clustering techniques, both nonparametric and parametric, will be executed within the R software environment. The project will apply the functional clustering methods to varved sediment data from lakes in Northern Sweden and Finland to infer seasonal climate changes over the last millennia.

Project 3: Advancing cosmic ray analysis: an open-source android neutron detector module

Our planet is continuously immersed in a dynamic flux of cosmic rays, spanning an extensive energy range from 10^7 eV/nucleon to 10^{20} eV/nucleon. Furthermore, the cosmic ray flux exhibits a remarkable variability, fluctuating from merely one particle per square meter and second to a solitary particle per square kilometre and century. The cosmic ray flux at Earth's orbital region is influenced by solar activity, particularly noticeable at energy levels below several tens of GeV/nucleon. This phenomenon positions temporal fluctuations in cosmic ray flux as invaluable indicators of solar activity. Ground-level instruments, such as neutron monitors, are pivotal in observing these fluctuations. Neutron monitors primarily capture secondary cosmic rays, predominantly neutrons, generated when cosmic rays interact with the Earth's atmosphere. This unique interplay between cosmic rays, solar activity, and Earth's environment has urged international collaborations to create global detector networks. The Neutron Monitor Database (NMDB) is a great demonstration of these efforts, with a key objective being global coverage. Achieving this goal entails the establishment of new monitoring stations to bridge the existing gaps. However, installing these stations can present some challenges, making cost-effective alternatives an attractive target. In light of these considerations, this project aims to conceive and rigorously evaluate an open-source Android application module designed to detect neutrons and monitor count rates from a compact neutron monitoring device. The aim is to perform a comprehensive comparative analysis using several statistical approaches, including penalized regression techniques. This analysis will also compare the performance of the proposed neutron detector with some of the existing NMDB neutron monitors.

Mathematical Foundations of Artificial Intelligence

Axel Flinth

Axel Flinth is an Assistant Professor in Mathematics. His research concerns different kinds of machine learning, i.e. how we with the help of mathematics can help computers find patterns in large sets of data. He is especially interested in compressed sensing, which concerns methods utilizing structrial assumptions to aid reconstruction of signals from incomplete data, and equivariance in deep neural networks, which studies how symmetries in the data can be utilized, and how what effect these methods have.

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Project 1: Equivariance and augmentation

In machine learning, the goal is often to detect symmetries 'hidden' in the data. In some tasks, these are however appearant a-priori. As an example, when classifying images of hand-written digits, the classification should not change if the digit is shifted in either x or y-direction. This task is hence *invariant* to translations. If the task on the other hand is to detect *where* the digit is located, the prediction should translate with a translation of the image. Such tasks are *equivariant*.

Under the umbrella of *Geometric Deep Learning*, researchers have come up with methods to, given a group of symmetries, design special types of so called neural networks that are guaranteed to be equivariant. Put simply, the strategy is to restrict the way the linear parts of the networks can be chosen. A more organic approach is to artificially inject the symmetries into the data via so-called *augmentation*. In this project, the aim is to understand the similarities and differences of these two approaches – in particular in the dynamics of the training of the networks. An internship could both involve studying theoretical issues, as well as performing experimental work.

Project 2: Total variation regularized optimization problems on measure spaces

Imagine an object characterized by a few point-like sources of different amplitudes situated somewhere in a continuous domain $\Omega \subseteq \mathbb{R}^d$ for some d. One can for instance think of a few bright stars on a night sky, or a collection of small colonies of luminescent bacteria on a petri dish. Such objects can be faithfully modelled by a measure $\mu_0 = \sum_{i=1}^{s} c_i \delta_{x_i}$, where $c_i \in \mathbb{R}$ are the amplitudes, $x_i \in \Omega$ are the positions and δ is the Dirac delta distribution. Also imagine that we are given an indirect measurement $b = A\mu_0 + n \in \mathbb{R}^M$ of the object, where A is a linear map and n is a noise vector. Although A surely not is injective (it maps from an infinite dimensional space of measures into the finite-dimensional $\mathbb{R}^M$), μ_0 can still be recovered from b. The key here is that μ_0 has a very special structure in that it only consists of a few spikes. Therefore, μ_0 can be (approximately) recovered from b via the solution of optimization problems regularized by the total variation norm (which is the 'measure version' of the ℓ_1-norm in finite dimension). This project is about analyzing and developing new methods for solving such problems.