Midwinter meeting in discrete probability
January 11-12, 2023, Umeå

All talks are in room MC313 in the MIT-building

Wednesday

9.00-9.30 Istvan Tomon: Constructing boxes with randomness

Abstract: Given a family of axis-parallel boxes in $\mathbb{R}^d$ with no k pairwise disjoint members, at least how many points are needed to pierce all members? Denote this minimum by $f_d(k)$ in the worst case. It was an old problem (the first variant going back to 1965, and then reiterated many times) whether $f_d(k)/k$ can be arbitrarily large in any fixed dimension d. I will present a probabilistic construction showing that $f_d(k) \geq k(\log k)^{d-2-o(1)}$, which not only answers the previous question positively for $d \geq 3$, but also matches the best known upper bound up to double-logarithmic factors.

9.35-10.05 Mia Deijfen: Superconcentration, chaos and multiple valleys in first passage percolation

We consider a dynamical version of first-passage percolation on the d-dimensional integer lattice with i.i.d. edge weights, where edge weights are resampled independently in time. Let $T(n)$ denote the passage time from the origin to the site n steps along the first coordinate axis at time $t = 0$, and let $\mu(t)$ denote the expected overlap between the time minimizing paths at time 0 and $t > 0$. We show that a subdiffusive behaviour of $T(n)$ is equivalent to a chaotic behavior of the time minimizing paths, manifested in that $\mu(t) = o(n)$. Known bounds for $Var(T(n))$ thus imply that indeed $\mu(t) = o(n)$ for $t > 0$. As a consequence we show that there are many almost disjoint paths with almost optimal passage time. This gives evidence that earlier work by Sourav Chatterjee for certain Gaussian disordered systems reflects a more general principle.

10.05-10.30 Coffee

10.30-11.15 Jason Behrstock: : Random graphs and applications to Coxeter groups

Abstract: This talk will serve as an introduction to a beautiful connection between probabilistic combinatorics and geometric group theory which is a fertile area for future work. Erdos and Renyi introduced a model for studying random graphs of a given density and proved that there is a sharp threshold at which lower density random graphs are disconnected and higher density ones are connected. Motivated by ideas in geometric group theory we will explain some new threshold theorems for random graphs and applications of these results to the geometry of Coxeter groups. This talk will include joint work with Falgas-Ravry, Hagen, Sisto, and Susse (in various combinations).

11.20-11.50 Vilhelm Agdur: Universal lower bound for community structure of sparse graphs

Lunch 12.00-13.15

13.15-13.45 Tom Britton: An epidemic on a social network with adaptive dynamics

Recent mathematical models for the spread of infectious diseases often take the social structures in a community into account by modelling the spread on a socially structured network. This network is most often considered fixed, or time varying independently of the disease. It has however become evident in recent epidemics (e.g. Ebola and Covid-19) that the underlying network is altered as a consequence of the epidemic by not yet infected people taking preventive measures breaking or moving ties to infected neighbours. In the talk we present such a model and show that 1) such preventive measures may in fact have a negative effect on a population level, leading to more infections, and 2) the model may have a discontinuous phase transition of the final fraction infected as a function of the transmission rate.

13.50–14.20 Joel Danielson: Minimum weight spanning structures in random (hyper-) graphs: Matchings, factors, spheres and more

14.25-14.55 Jonas Sjöstrand: Monotone subsequences in random permutations

The Robinson-Schensted correspondence gives a bijection between permutations of $1, 2, \dots, n$ and pairs of standard Young tableaux of the same shape. A celebrated theorem by Vershik and Kerov and, independently, by Logan and Shepp in the 1970s states that for a uniformly random permutation the corresponding Young diagram converges to a limit shape (under a suitable scaling) as n tends to infinity. By a theorem of Greene, the sum of the lengths of the longest k rows in the Young diagram equals the largest possible cardinality of a union of k increasing subsequences in the permutation, so the Vershik-Kerov-Logan-Shepp result can be viewed as a statement about sizes of increasing subsequences.

We will present generalizations of this result in two directions: First, we will allow for random permutations that are only *locally* uniform: Draw n points independently from some absolutely continuous distribution ρ on the plane and interpret them as a permutation by mapping i to j if the ith point from the left is the jth point from below. Second, we will care not only about the *size* of the increasing subsequences but also their *location*.

15.00-15.20 Coffee

15.20-15.50 Paul Thévenin: Local limits of descent-biased trees

15.55-16.15 Fabian Burghart: Obtaining polynomial invariants from destroying trees

16.20–end Per Håkan Lundow: Damage spreading in the random-cluster model for 2D and 3D grid graphs.

18.30 Dinner at Köksbaren for registered participants

Thursday

9.00-9.30 Victor Falgas-Ravry: 1-dependent percolation on high dimensional integer lattices

Consider a random subgraph of the square integer lattice obtained by including each edge independently at random with probability p, and leaving it out otherwise. The Harris–Kesten theorem states that if p is at most 1/2, then almost surely all connected components in this random subgraph are finite, while if p¿1/2 then almost surely there exists a unique infinite connected component.

But now what if we introduced some local dependencies between the edges? More precisely, suppose each edge still has a probability p of being included in our random subgraph, but its state (present/absent) may depend on the states of edges it shares a vertex with. To what extent can we exploit such local dependencies to delay the appearance of an infinite component? This problem, which originates in work of Balister, Bollobás and Walters in 2005 on continuum percolation, remains largely open. In this talk, I will survey what is known and discuss some recent progress on the analogous problem on the d-dimensional integer lattice.

9.35-10.05 Annika Heckel: The hitting time of a clique factor

10.05-10.30 Coffee

10.30-11.00 Svante Janson: Asymptotic normality of the size of the giant component in random bipartite graphs and intersection graphs.

Consider a bipartite random graph $G(n, m, p)$, with $n + m$ vertices, and $p = \lambda/\sqrt{nm}$. The corresponding random intersection graph is defined as the left side of $G(n, m, p)$, with two vertices adjacent if they have a common neighbour in $G(n, m, p)$.

Let $n, m \to \infty$. It is well known that if $\lambda > 1$, then there is whp a giant component in the random bipartite graph and in the intersection graph. It was recently shown by Dong and Hu (2022) that the size of the giant in the intersection graph is asymptotically normal, at least in some range of n and m. We extend this to all n and m, and show, more generally, that in $G(n, m, p)$, the sizes of the left and right parts of the giant have a joint asymptotic normal distribution; the 2-dimensional asymptotic distribution is non-singular if $m = \Theta(n)$, but singular otherwise.

Joint work with Oliver Riordan.

11.05-11.35 Christoffer Olsson: Automorphisms of random trees

Counting objects up to symmetry is a classical subject in combinatorics. In this talk, we take a probabilistic viewpoint and study the automorphism group of two types of random trees: Galton–Watson trees (a family of random rooted trees that include, for example, plane, binary and labelled trees) as well as

Pólya trees (rooted, unordered and unlabelled trees). Specifically, we prove that, in both cases, the size of the automorphism group follows a log-normal distribution, asymptotically as the size of the tree goes to infinity. Our proofs use both probabilistic tools and methods from analytic combinatorics.

11.35–11.55 Fiona Skerman: Quasirandom-forcing tournaments

A tournament H is quasirandom-forcing if the following holds for every sequence $(G_n)_n$ of tournaments of growing orders: if the density of H in G_n converges to the expected density of H in a random tournament, then $(G_n)_n$ is quasirandom. Every transitive tournament with at least 4 vertices is quasirandom-forcing, and Coregliano et al. [Electron. J. Combin. 26 (2019)] showed that there is also a non-transitive 5-vertex tournament with the property. We show that no additional tournament has this property. This extends the result of Bucic et al. [Combinatorica 41 (2021)] that the non-transitive tournaments with seven or more vertices do not have this property.

This is joint work with Robert Hancock, Adam Kabela, Dan Král', Taísa Martins, Roberto Parente and Jan Volec.

Lunch 12.00-13.15

13.15-13.45 Matteo Sfragara: Competing first passage percolation on the configuration model

In this talk we will discuss competing first passage percolation on graphs generated by the configuration model, where two infection types compete to invade the vertices in the graph. Initially, two uniformly chosen vertices are infected with type 1 and type 2 infection, respectively, and the infection then spreads via nearest neighbors. The time it takes for the type 1 (resp. 2) infection to traverse an edge e is given by a random variable $X_1(e)$ (resp. $X_2(e)$) and, if the vertex at the other end of the edge is still uninfected, it then becomes type 1 (resp. 2) infected and immune to the other type. We first introduce the mathematical model, assuming that the degrees follow a power-law distribution with exponent τ, and present some known results in the case of finite variance ($\tau > 3$) and in the case of finite mean but infinite variance ($2 < \tau < 3$.) We then focus on the case of infinite mean ($1 < \tau < 2$) and show that, with high probability as the number of vertices n tends to infinity, one of the infection types occupies all vertices except for the starting point of the other type. Moreover, both infections have a positive probability of winning regardless of the passage times distribution. The result is also shown to hold for the erased configuration model and when the degrees are conditioned to be smaller than n^α for some $\alpha > 0$. This talk is based on joint work with Mia Deifen (Stockholm University) and Remco van der Hofstad (Eindhoven University of Technology).

13.50-14.20 Colin Desmarain: Depths in random recursive metric spaces

As a generalization of random recursive trees and preferential attachment trees, we consider random recursive metric spaces. These spaces are constructed from random blocks, each a metric space equipped with a probability measure, containing a labelled point called a hook, and assigned a weight. Random recursive metric spaces are equipped with a probability measure made up of a weighted sum of the probability measures assigned to its constituent blocks. At each step in the growth of a random recursive metric space, a point called a latch is chosen at random according to the equipped probability measure and a new block is chosen at random and attached to the space by joining together the latch and the hook of the block.

We prove a law of large numbers and a central limit theorem for the insertion depth, the distance from the master hook to the latch chosen. A classic argument proves that the insertion depth in random recursive trees is distributed as a sum of independent Bernoulli random variables. We generalize this argument by approximating the insertion depth in random recursive metric spaces with a sum of independent random variables.

14.25-14.55 Klas Markström: From random K-SAT to hypergraph transversals

15.00 Coffee