Curvature

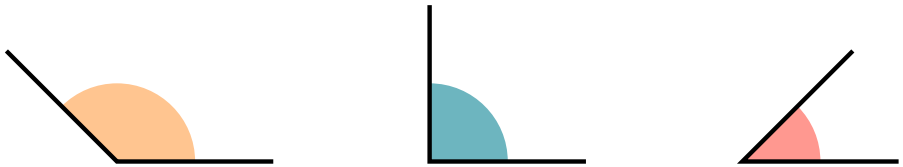
Noémie Vlaanderen Oldenzeel

22/04/2026

Question

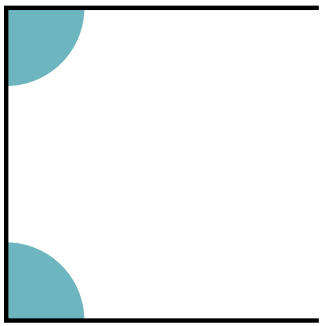
Right angle (rät vinkel)

90° angle



Question 1

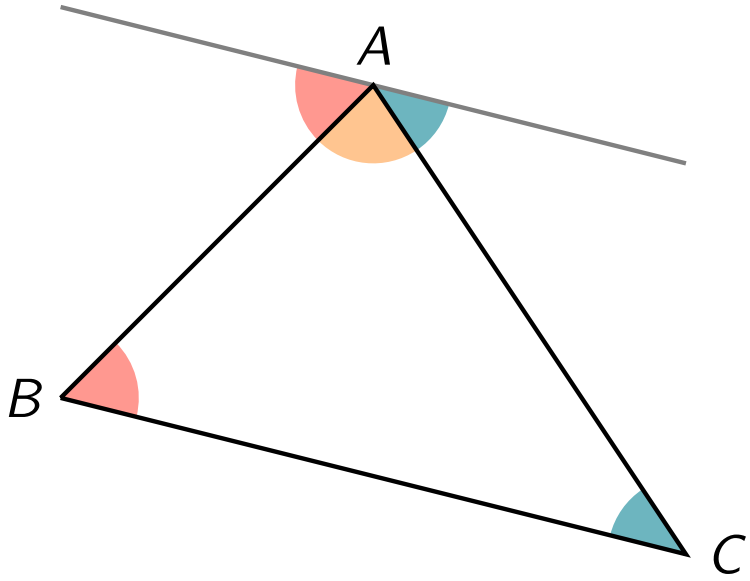
Can you draw a triangle with three right angles?



Triangle in the plane

Theorem

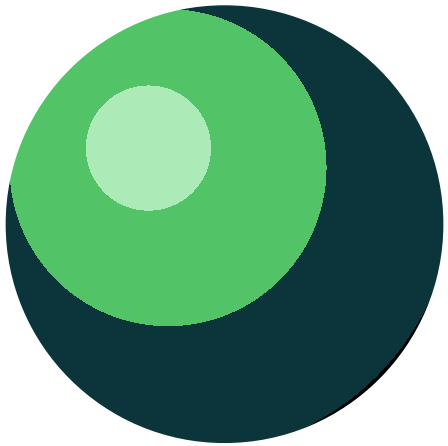
The sum of the angles of a triangle in a plane is $180°$



Sphere

Unit sphere

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$



Question 2

What should count as a line on a sphere?

Shortest path

https://gisgeography.com/great-circle-geodesic-line-shortest-flight-path/

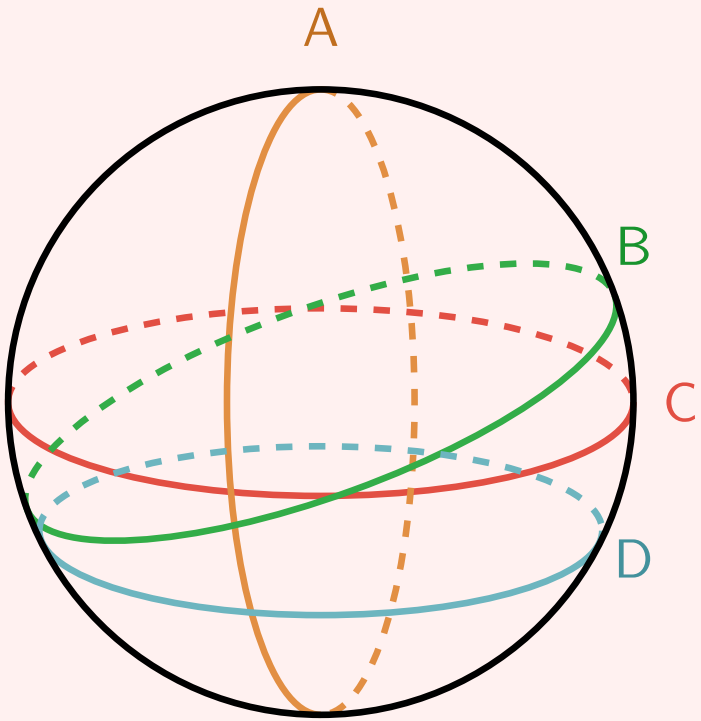
Great circles

Great circles

Circles dividing sphere in two equal parts

Question 3

Which of these are great circles?



Lines on a sphere

Lines

The lines on a sphere are great circles.

Question 4

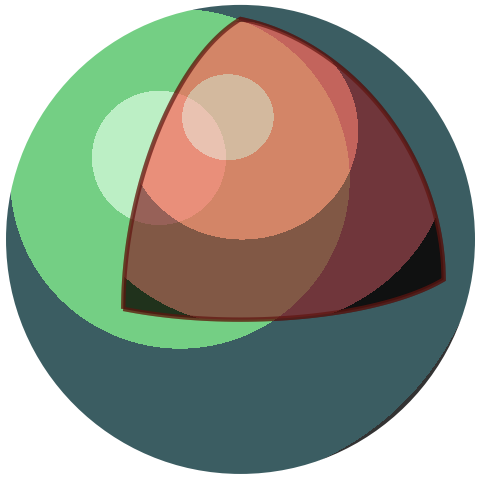
True or false: in the plane or on a sphere:

1. Suppose we have two points, there exist a unique line thought the points.
2. Suppose we have a line and a point not on the line. There is a unique parallel line through the point.
3. Two distinct lines meet in at most one point.
4. Any three distinct lines form a triangle.
5. Any three distinct lines that do not meet in a single point form a triangle.

Question

Question 1

Can you draw a triangle with three right angles?



Question 5

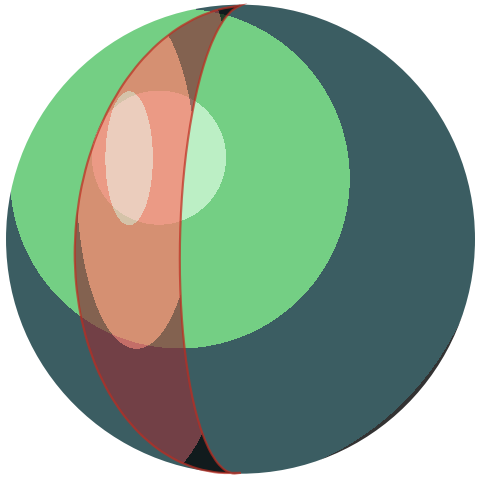
What can you say about the sum of the angles of a triangle on a sphere?

1. What is the smallest this sum can be?
2. What is the biggest this sum can be?

Biangles

Biangle

A two-sided polygon



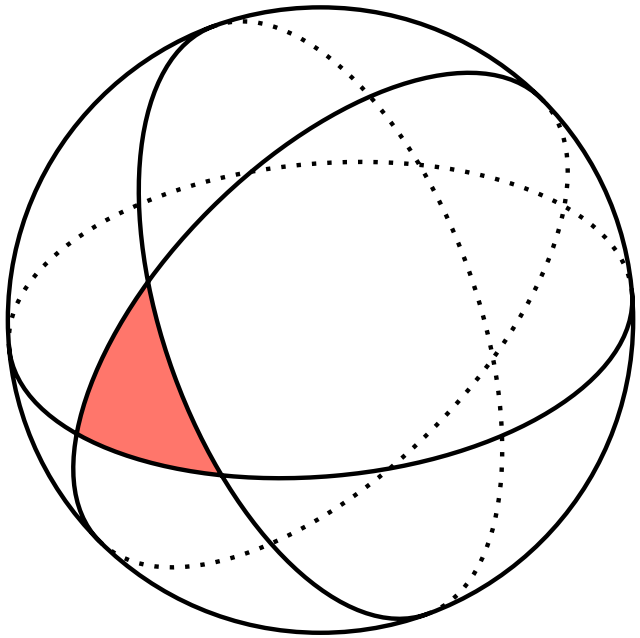
Lemma

- Both angles in the biangle are the same.
- Consider a biangle with angles θ (in degrees), then the area of the biangle is $4\pi\frac{\theta}{360}$

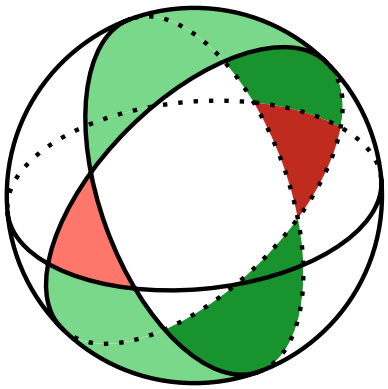
Area of a triangle

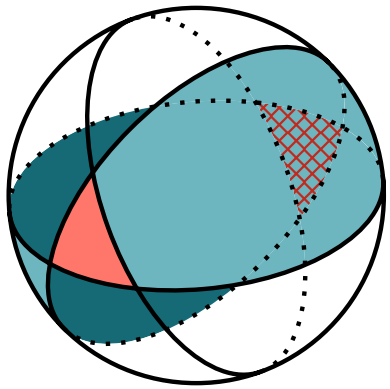
Question 6

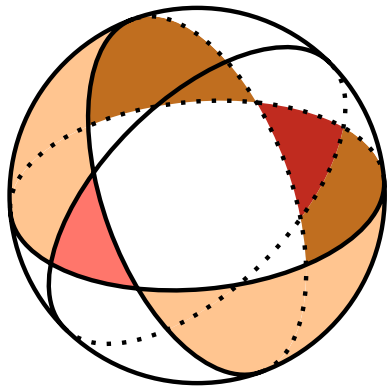
Let T be a triangle on a sphere with angles α, β, γ. Show that the area of T is $\frac{2\pi}{360}(\alpha + \beta + \gamma) - \pi$



Area of a triangle: proof







Curvature

Plane	Sphere
Lines	Great circles
Sum of angles in a triangle $= 180^\circ$	Sum of angles in a triangle $> 180^\circ$
Flat	Positively curved

Question 7

Can you think of an object where there is a triangle with the angles summing to $< 180^\circ$?