Problem session 3

Session problems

Try problems 4.5, 4.3, 4.7, 4.6 and 4.26 from the book — we will put them up on the board.

Homework problems

You may submit your written solutions until the next meeting (12 December) in person, or by email (victor.falgas-ravry@umu.se).

Problem 1. Santa Claus has an infinite stable of alpha reindeers, who are unpleasant and aggressive, and an infinite stable of beta reindeers, who are agreeable and peace-loving. Further, Santa Claus has a sleigh which this year is pulled by 2023 reindeers all in a line. However there are some constraints on the line of reindeers: an alpha reindeer cannot be place behind another alpha reindeer, as otherwise chaos will ensure. Show that there are precisely

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{2025} - \left(\frac{1-\sqrt{5}}{2}\right)^{2025}\right)$$

different ways of arranging alpha and beta reindeers along the line while respecting this constraint.

Problem 2. Let the sequence u_n be defined recursively by the initial values $u_0 = 3$, $u_1 = 2$, and by the recurrence relation $u_{n+2} = (u_{n+1})^2/u_n$ for all integers $n \geq 0$. Give a closed-form expression for u_n for general n (i.e. an explicit non-recursive formula for u_n and prove that it is correct.

Problem 3. Let n be a natural number. Suppose we are given 3^n coins, one of which is counterfeit and lighter than the others. Further, we have access to a scale, which may be used to determine which of two disjoint sets of coins is the heavier one or whether they have the same weight. Prove the best upper bound you can on the number of weightings needed to identify the counterfeit coin.