

Problem 1. Suppose that l is a perpendicular bisector for a line-segment AB . Prove that M belongs to l if and only if $|AM| = |BM|$.

Problem 2. Consider the configuration in the problem 9.8 from the book. Determine the angle $\angle H_B H_C B$ in terms of the angles of the triangle ABC .

Problem 3. Let ABC be a triangle. Prove that the three perpendicular bisectors of ABC intersect at a point O . Furthermore, prove that $|OA| = |OB| = |OC|$. (The last observation implies that the points A , B and C are on the same circle whose center is O .)

Problem 4. Suppose that ABC is a triangle chosen so that $|AB| = |AC|$. Let l be a line going through the point A chosen so that l is the angle-bisector for the angle $\angle CAB$. Prove that l is also an altitude, median and a perpendicular bisector.

Problem 5. Let $ABCD$ be a parallelogram. Suppose that the diagonals AC and BD are perpendicular to each others. Prove that $ABCD$ is in fact a rhombus, that is, prove that all four sides of $ABCD$ have equal lengths.

Problem 6. Let ABC be an acute triangle. Prove that the three altitudes of ABC intersect at a point. (Hint: draw lines l_a , l_b and l_c going through points A , B and C so that they are parallel to the sides BC , AC and AB respectively.)

Problem 7. Let A , B and C be four points in the plane. Prove that the following three conditions are equivalent. (You might have seen before that these are the different ways to determine whether $ABCD$ is a parallelogram. Your task is to show that these different conditions are indeed equivalent.)

1. $AB \parallel CD$ and $BC \parallel AD$
2. $\angle BAD = \angle DCB$ and $\angle CBA = \angle ADC$
3. $|AB| = |CD|$ and $|BC| = |AD|$

Problem 8. Determine the size of the angle x in the following figure.

