Problems 23^{rd} April.

Figures can be found on the second page.

1. Suppose that A, B and X are chosen on the same circle so that AB is a diameter of the circle. Prove that $\angle BXA = 90°$.

2. Suppose that $ABCD$ is a cyclic quadrilateral. Prove that $\angle CBA + \angle ADC = 180°$ and $\angle BAD + \angle DCB = 180°$.

3. Uppgift 10.1

4. Uppgift 10.2

5. Uppgift 10.10

6. The goal of this problem is to give a proof of the fact that if $\angle CAB$ is an inscribed angle, then the corresponding central angle $\angle COB$ satisfies $\angle COB = 2 \cdot \angle CAB$.

Step 1 First prove the statement when AC is a diameter of the circle. (Hint: try to prove that $\angle OAB = \angle BOA$).

Step 2 Now suppose that the center O is contained inside the angle $\angle CAB$: that is, when one draws the line-segments AC and AB, the point O is between these line-segments.

Let M be a point chosen so that AM is a diameter of the circle. By considering the angles $\angle CAM$ and $\angle MAB$, try to finish the proof.

Step 3 Finally suppose that O is not contained inside the angle $\angle CAB$. and without loss of generality assume that the angle $\angle CAB$ is fully contained in the half-circle bounded by the diameter drawn at the point A (see the picture).

Let M be the point chosen as before. Complete the proof of this case by using the fact that $\angle CAB = \angle MAB - \angle MAC$.

7. Uppgift 10.7

8. Let A, B and C be points on the same circle. Let $M \neq A$ be a fourth point on the circle chosen so that the line BM bisects that angle $\angle CAB$, i.e. $\angle MAB = \angle CAM$.

   (i) Prove that $|MB| = |MC|$.

   (ii) Let I be a point in the interior of the line-segment AM chosen so that $|MB| = |MI| = |MC|$. Prove that $\angle ABI = \angle IBC$, i.e. that I also bisects the angle $\angle ABC$.

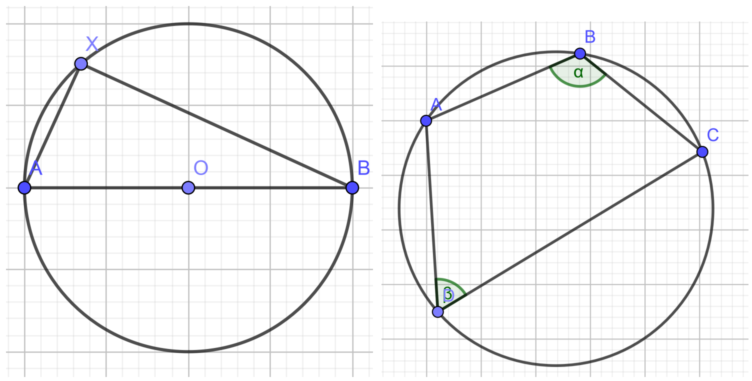

Figure 1: Figures for Problems 1 and 2

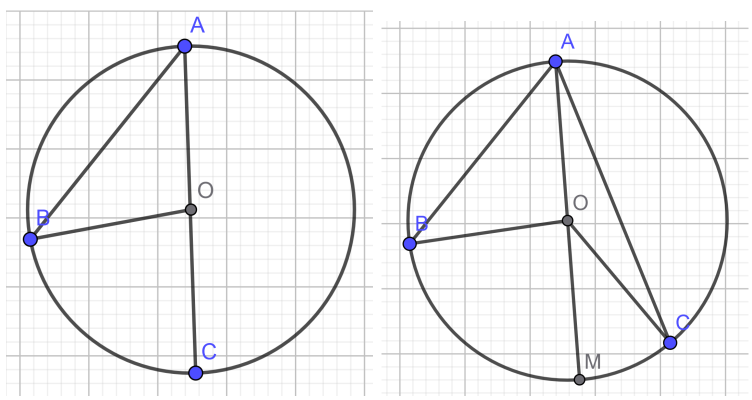

Figure 2: Figures for Step 1 and Step 2 in Problem 6

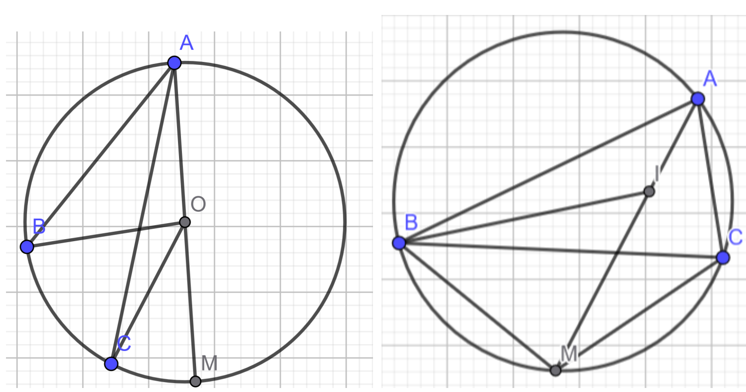

Figure 3: Figure for Step 3 in Problem 6 and Problem 8