



Formalisation of mathematics in Lean 4 (Formalisering av matematik i Lean 4)

Credits: 7.5 ECTS

Course period: Autumn 2026

Main field of study: Mathematics/Computer Science

Progress level: PhD

Grading scale: Pass/Fail

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Description

Formal proof-checkers are quickly becoming incorporated into scientific practices associated to mathematical research, in large part due to the rise of digital tools like large language models. In this digital wave, the programming language Lean 4 and the accompanying mathematical library Mathlib have emerged as the industry standard for formally verifying mathematical statements. This course is designed to be an introductory reading course to build familiarity with Lean 4 and Mathlib through hands-on code reading and writing. Throughout the course, students will learn how to work with Lean 4 and Mathlib to formalise mathematical statements, define new mathematical structures, and write proofs.

Prerequisites

This course is aimed at doctoral students in mathematics or computer science. The only prerequisites are familiarity with undergraduate level mathematics and basic computer programming skills.

Objective

The course aims to provide a practical foundation in the use of formal proof assistants in mathematics, with emphasis on how mathematical definitions, statements and proofs are expressed in Lean 4 and how the Mathlib library is used to build and verify them.

Learning outcomes

After following the course, the students should be able to:

- read and interpret formal statements and definitions in the syntax of Lean 4,
- look up definitions, lemmas and theorems in Mathlib using various tools,
- define their own mathematical structures and formalise their own mathematical statements in Lean 4,
- understand and write simple proofs using tactics and proof terms.

Form of instruction

The first part (5 weeks) of the course will be conducted as a reading course. Students will engage in self-study of the reading material and participate in scheduled meetings to discuss the content. This will be done by reading the chapters and working through the coding worksheets of the textbook *Mathematics in Lean*. In the second part (3 to 4 weeks), students will do individual or group work to produce their own examples of Lean formalisations and discuss these in oral presentations during regularly scheduled meetings.

Content

The course introduces the use of a formal proof assistant in mathematical practice, with an emphasis on reading, writing and verifying formal proofs in Lean 4 and on navigating the Mathlib library. The first part follows the textbook *Mathematics in Lean*, progressing from the basic syntax of terms, types and tactics through logic to



the formalisation of mathematical structures. The second part is devoted to independent work in which students formalise definitions and statements of their own choosing. Course topics include:

- searching Mathlib: naming conventions, documentation and look-up tools
- logic: quantifiers, connectives and their proof rules
- sets, functions and elementary number theory
- structures, type classes and the Mathlib algebraic hierarchy

Examination

The examination will consist of an oral presentation delivered by the student which demonstrates their understanding of Lean 4 code and covers one of the following.

- A walkthrough of a chapter of *Mathematics in Lean*.
- Individual or group work leading to a concrete example of a Lean 4 formalisation of an interesting mathematical statement.

Literature

The principal reading material for the course will be the textbook *Mathematics in Lean* by Jeremy Avigad and Patrick Massot. The text is available at https://leanprover-community.github.io/mathematics_in_lean/ for free, along with instructions on how to install Lean 4 and download the accompanying coding worksheets. Other useful resources include:

- *Theorem proving in Lean 4*, by Jeremy Avigad, Leonardo de Moura, Soonho Kong, Sebastian Ullrich with contributions from the Lean Community (freely available online)
- Kevin Buzzard's lecture notes titled *Formalising mathematics* (freely available online)
- Mathlib documentation, https://leanprover-community.github.io/mathlib4_docs/index.html
- Loogle, an online Mathlib look-up website, <https://loogle.lean-lang.org/>