Optimization on Manifolds

Credits: 7.5 ECTS

Course organizer and lecturer

Department of Mathematics and Mathematical Statistics
Per Åhag, e-mail: per.ahag@umu.se

Institute of Mathematics, EPFL
Nicolas Boumal, e-mail: nicolas.boumal@epfl.ch

Course period: Spring semester 2021

Prerequisites

Students are expected to have a basic knowledge of Analysis, Linear algebra, Elements of numerical linear algebra and numerical methods, and Programming skills in a language suitable for scientific computation (Matlab, Python, Julia...).

Objective

The objective is to develop, analyze and implement numerical algorithms to solve optimization problems of the form: min f(x) where x is a point on a smooth manifold. To this end, we first study differential and Riemannian geometry (with a focus dictated by pragmatic concerns). We also discuss several applications.

There will be one two-hour lecture per week, and one two-hour exercise session including both theoretical work and programming assignments per week.

Content

This course covers the following topics:

- Applications of optimization on manifolds
- First-order Riemannian geometry in Euclidean spaces
- First-order optimization algorithms on manifolds
- Second-order Riemannian geometry in Euclidean spaces
- Second-order optimization algorithms on manifolds
- Fundamentals of differential geometry (general framework)
- Riemannian quotient manifolds
- More advanced geometric tools
- Geodesic convexity

Learning Outcomes:

By the end of the course, the student must be able to:

- Manipulate concepts of differential and Riemannian geometry.
- Develop geometric tools to work on new manifolds of interest.
- Recognize and formulate a Riemannian optimization problem.
- Analyse implement and compare several Riemannian optimization algorithms.
- Apply the general theory to particular cases.
- Prove some of the most important theorems studied in class.

Examination

The examination consists of a written exam and assignments during the course.

Literature

1) Lecture notes: “An introduction to optimization on smooth manifolds”, available online: http://www.nicolasboumal.net/book
2) Book: “*Optimization Algorithms on Matrix Manifolds*”, P.-A. Absil, R. Mahoney and R. Sepulchre, Princeton University Press 2008
3) Book: “*Introduction to Smooth Manifolds*”, John M. Lee, Springer 2012
4) Book: “*Introduction to Riemannian Manifolds*”, John M. Lee, Springer 2018