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Manifold-like Polyfolds (Mångfaldsliknande Polyfolder)

Credits: 7.5 ECTS
Course Period: February 2026 – June 2026
Main field of study and progress level: *Mathematics*, third cycle
Grading scale: Pass/fail
Course coordinator: Fredrik Ohlsson fredrik.ohlsson@umu.se
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Description:
This course is designed as an introductory course in M-polyfold theory. We introduce sc-Banach spaces and sc-smooth maps, the role of sc-smooth retractions/retracts as local models, and the resulting notion of an M-polyfold (a generalization of manifolds allowing varying local dimension). The course then develops the resulting geometrical framework, including implicit function theorems, compactness and perturbation/transversality results, and orientations and invariants.

Prerequisites:
A solid background in functional analysis and differential geometry (at the level of Banach manifolds). A good knowledge of topology is recommended.

Objectives:
The expected learning outcomes of the course are as follows. After completing this course, students should be able to:

- Explain sc-structures, sc-differentiability, and the chain rule in sc-calculus.
- Work with sc-smooth retractions/retracts and construct basic examples of M-polyfolds.
- Define strong bundles over M-polyfolds and state the notion of an sc-Fredholm section.
- Apply the main local results (basic germs, contraction germs, implicit function theorems) to describe solution sets.
- Describe the Fredholm package in M-polyfolds: auxiliary norms, compactness, perturbations/transversality, and orientations.

Contents:

- Sc-calculus: sc-structures, sc-smoothness, chain rule, and boundary recognition
- Retracts and M-polyfolds: sc-smooth retracts, tangent construction, degeneracy index, and boundary geometry
- Tame M-polyfolds and strong bundles
- Basic sc-Fredholm theory: germs, basic germs, and implicit function theorems
- Manifolds and strong retracts; smooth finite-dimensional submanifolds; sc-differential forms
- The Fredholm package: auxiliary norms, compactness, perturbation theory, and transversality
- Orientations and invariants for proper sc-Fredholm sections

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Form of instruction:
The course will be conducted as a reading course. Students will engage in self-study of Part I (Chapters 1-6) and participate in scheduled meetings.

Examination:
The examination consists of active participation and several oral presentations on the assigned reading. In addition, each student will complete an extensive project under the mentoring of the course coordinator.

Literature:
The primary reading material for the course is
Polyfold and Fredholm Theory (Part I: Basic Theory in M-Polyfolds)
by Helmut Hofer, Krzysztof Wysocki, and Eduard Zehnder
Springer, 2021
https://doi.org/10.1007/978-3-030-78007-4