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Modular forms (Modulära former)

Credits: 7.5 ECTS
Course Period: October 2025 – March 2026
Main field of study and progress level: *Mathematics*, third cycle
Grading scale: Pass/fail

Course coordinator: Henrik Gustafsson henrik.gustafsson@umu.se
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Description:
This course is designed as an introductory reading course for the theory of modular forms. These are functions on the complex upper half-plane satisfying specific transformation properties under the modular group $SL_2(\mathbb{Z})$.

The study of modular forms lies at the intersection of number theory, algebra, and analysis, with deep connections to elliptic curves, L-functions and arithmetic applications.

Through the course, students will learn both the foundational theory of modular forms and explore a topic of their choice in a short project on the applications of modular forms.

Prerequisites:
The student should have a strong foundation in complex analysis, basic algebra and linear algebra.

Objectives:
The expected learning outcomes of the course are as follows. After completing this course, students should be able to:

- Define fundamental concepts such as the definition of modular forms and their properties.
- Recognize and illustrate examples of common modular forms, including Eisenstein series and the discriminant function.
- Compare and apply standard techniques to analyze modular forms.
- Evaluate the role of modular forms in their applications to number theory and related areas.

Content:
- Modular forms
- Elliptic and modular curves
- Modular curves as Riemann surfaces
- Dimension formulas
- Eisenstein series
- Hecke theory
- Applications

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Form of instruction:
The course will be conducted as a reading course. Students will engage in self-study of the reading material and participate in scheduled meetings to discuss course content moderated by the course coordinator. In addition, each student will complete a short project under the supervision of the course coordinator on a chosen topic related to modular forms and their applications.

Examination:
The examination consists of an oral exam based on the reading material and a presentation of the short project. Successful completion of both components is required to pass the course.

Literature:
The primary reading material for the course is
A First Course in Modular Forms
by Fred Diamond and Jerry Shurman
https://doi.org/10.1007/978-0-387-27226-9

Available as PDF from the university library.

Students are expected to seek out further literature for their chosen projects.