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An Extended EDAS Method with Circular Intuitionistic Fuzzy Value Features and Its Application to Multi-criteria Decision-making Process

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Contents

Fuzzy logic is a generalization of classical Boolean logic. The basis of fuzzy logic is based on fuzzy sets and subsets.

In Boolean logic, an element either belongs to a set or it does not. This state of belonging can be characterized by χ characteristic function

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

for all $x \in X$ and $A \subset X$.

However, we encounter situations of uncertainty rather than certainty in real life. The reason why we encounter these ambiguous situations is that the language and linguistic variables have an ambiguous structure.

Some linguistic variables:

1. Age
2. Height of people
3. Price of objects
4. Emotions of people, etc.

Examples

Example

Consider that we are going to buy a car. If we describe cars over €10,000 as **"expensive"** and cars under €10,000 as **"cheap"** in terms of Boolean logic, as a result of these two propositions, we can call both cars priced at €11,000 and €100,000 expensive. Or we can call both cars priced at €9,000 and €1000 cheap.
However, this cannot be said to be very useful in daily life.

Is he tall or short?

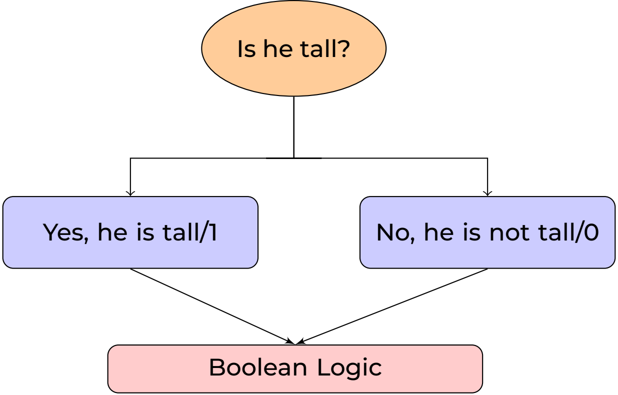
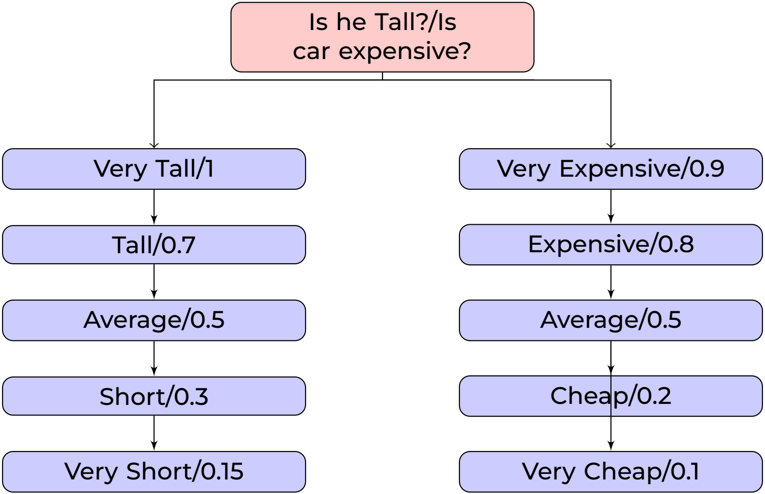


Figure: Characterization of Boolean logic

Fuzzy logic is based on a gradation in the [0,1] range, breaking the 0-1 precision of Boolean logic.



The Membership Function

Just as the concept of certainty can be characterized mathematically with the help of the χ function in classical set theory, the uncertainty situation can be characterized with the help of the membership function in fuzzy set theory. In order to model uncertain and uncertain situations mathematically, Zadeh proposed fuzzy logic and fuzzy set theories in 1965 (Zadeh 1965).

Definition

A fuzzy set A of a given set X is defined with a function which is called the membership function of A and it is denoted by

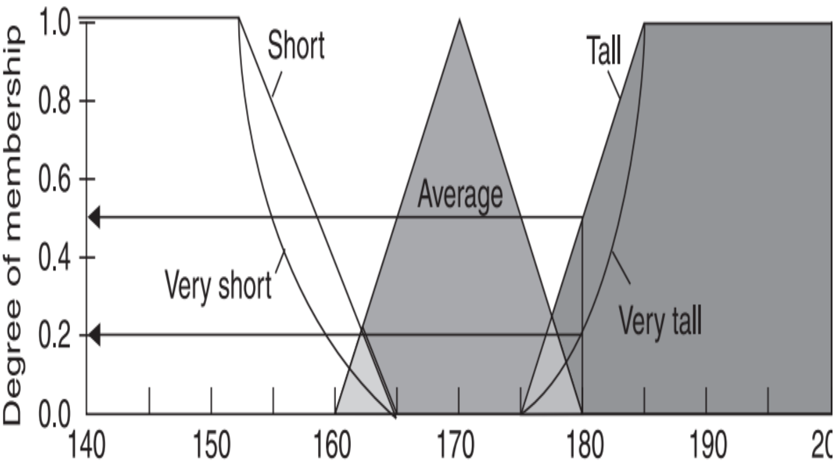
$$A = \{< x, \mu_A(x) > | x \in X\} .$$

The value $\mu_A(x)$ is said to be the grade of membership of the element x to the set.

Example

Example

The universe of discourse, men's heights – consists of five fuzzy sets: very short , short , average , tall and very tall . For example, a man 180 cm tall is a member of the tall set with a degree of membership of 0.5 and a member of the very tall set with a degree of membership of 0.2 (Guo 2013).


Degree of membership
1.0
0.8
0.6
0.4
0.2
0.0
140
150
160
170
180
190
20
Short
Very short
Average
Tall
Very tall

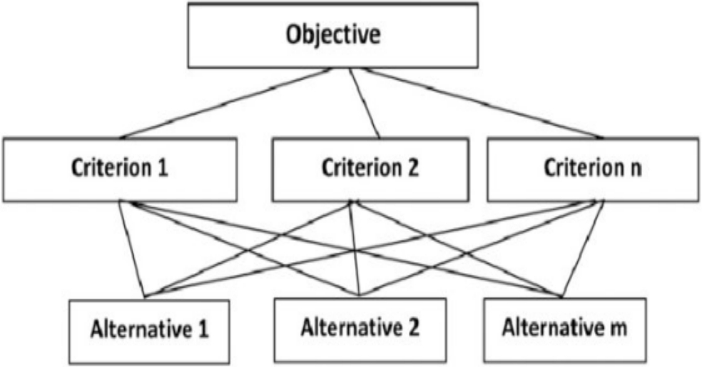
Fuzzy logic **(fuzzy set theory, fuzzy measure and integral theory)**, which aims to solve real-life problems by modeling the uncertainties arising from the partial membership of an element to a set, has been generalized over time depending on the process of modeling uncertain information.

This is a concept that has applications in many different fields such as economics, engineering, decision-making and management with several real life problems like

- pattern recognition,
- classification and clustering,
- decision making problems like selection problem,
- medical diagnosis etc.

What is multi-criteria decision making?

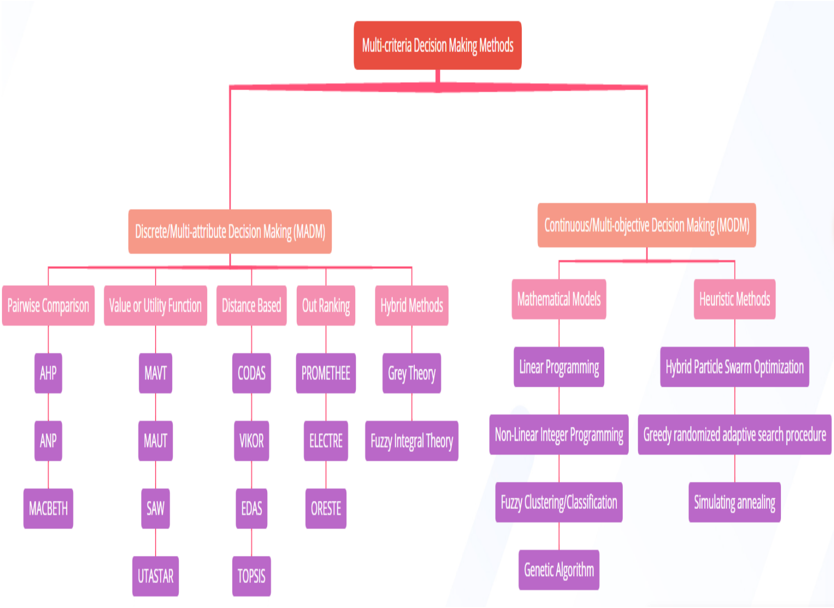
Multi-criteria decision making is the process of selecting the most appropriate alternative among alternatives according to conflicting criteria.



In the multi-criteria decision-making process, decision makers

1. evaluate each alternative under conflicting criteria and rank the alternatives from best to worst.
2. aim to reach the **"optimal choice/alternative"** by taking advantage of the fuzzy set logic technique's ability to model uncertain, inconsistent and incomplete information.

In this process, the decision maker can use some decision methods in accordance with the structure of the problem:



Multi-criteria Decision Making Methods
Discrete/Multi-attribute Decision Making (MADM)
Continuous/Multi-objective Decision Making (MODM)
Pairwise Comparison
Value or Utility Function
Distance Based
Out Ranking
Hybrid Methods
Mathematical Models
Heuristic Methods
AHP
ANP
MACBETH
MAVT
MAUT
SAW
UTASTAR
CODAS
VIKOR
EDAS
TOPSIS
PROMETHEE
ELECTRE
ORESTE
Grey Theory
Fuzzy Integral Theory
Linear Programming
Non-Linear Integer Programming
Fuzzy Clustering/Classification
Genetic Algorithm
Hybrid Particle Swarm Optimization
Greedy randomized adaptive search procedure
Simulating annealing

Relations

Fuzzy sets and logic, uncertainty, and multi-criteria decision-making are concepts that are closely related to the field of artificial intelligence (AI), especially in the context of explainable AI (XAI) and machine learning.

Relation to AI: Fuzzy logic allows for reasoning under uncertainty by handling imprecise information. It is used in AI for decision-making in systems where information is not strictly binary (true or false), providing a more flexible and human-like approach to processing information. Uncertainty is a fundamental aspect of AI, as real-world situations often involve incomplete or ambiguous information. AI models need to handle uncertainty to make informed decisions.

Relation to XAI and ML: In XAI, fuzzy logic can enhance interpretability by allowing models to express uncertainty and deal with imprecise data. It enables more transparent decision-making, making it easier for humans to understand and trust AI systems. In machine learning, fuzzy systems are applied for tasks involving uncertainty and vagueness, contributing to model interpretability. Dealing with uncertainty is crucial for XAI, where providing explanations for model predictions becomes challenging in uncertain environments. In machine learning, techniques such as probabilistic models and Bayesian approaches address uncertainty, contributing to both model interpretability and reliability.

Relation to AI: MCDM involves making decisions based on multiple criteria or objectives. In AI, MCDM is applied to complex decision-making scenarios where multiple factors need to be considered.

Relation to XAI and ML: In XAI, explaining decisions derived from MCDM models becomes essential for transparency. MCDM techniques are used in machine learning to handle complex decision spaces and enable models to make choices based on multiple criteria, contributing to interpretable and explainable models.

In summary, these concepts are integral to the broader field of AI, contributing to the development of more interpretable and understandable machine learning models. They play a crucial role in addressing challenges related to uncertainty, imprecision, and complex decision-making, which are essential considerations in both the development and explanation of AI systems.

What is aim of the presentation and its Structure?

Aim of this presentation is that to propose an MCDM ("multi-criteria decision-making") algorithm by utilizing C-IFS ("circular intuitionistic fuzzy sets") features, which is a more reliable tool to handle the uncertainties in the data.

- Firstly, I extend the theory of C-IFS and defines some algebraic aggregation operators based on Archimedean t-norms operations between the pairs of C-IFSs.
- Further, I define an EDAS ("Evaluation Based on Distance from Average Solution") method for MCDM problems using the subtraction and division operations.
- The performance of the stated MCDM algorithm is discussed through numerical examples and compared their study with the existing approaches.

What is missing in the literature?

1. There is no work on the EDAS method for C-IFSs.
2. Scores of average solutions are used rather than intermediate solutions while applying the EDAS method, and so the algebraic operations of scalars are enough to apply the EDAS method. This situation can be accepted as a limitation of the EDAS method.

So, I aim to remove this restriction of the EDAS method for C-IFSs by defining the subtraction and division operations for C-IFSs and circular intuitionistic fuzzy values (C-IFVs) with the help of t-norms and t-conorms.

- (Atanassov 1986), defined the concept of IFS ("Intuitionistic fuzzy set") in which a new degree named as non-membership function $\varphi \in [0, 1]$ is added along with membership function $\varsigma \in [0, 1]$ such that $\varsigma + \varphi \in [0, 1]$.

- (Atanassov 2020), expanded this concept to C-IFS ("circular intuitionistic fuzzy set") by considering the circles with center $(\varsigma_A(x), \varphi_A(x))$ instead of points. Each element in a C-IFS is represented by a circle whose center is $(\varsigma_A(x), \varphi_A(x))$ and radius $0 \leq r \leq 1$. In this C-IFS, the sum of the "degrees of memberships" within this circle is at most equal to one.

Namely, this concept expresses the elements of X with circles instead of points of the intuitionistic fuzzy environment and better models for ambiguous and inconsistent information than the concept of IFS. As a result, the idea of C-IFS allows decision-makers to define degrees as circular membership functions.

Definition

A C-IFV α is characterized by a membership degree $\varsigma_\alpha \in [0,1]$, a non-membership degree $\varphi_\alpha \in [0,1]$ and a radius $r_\alpha \in [0,1]$ with $\varsigma_\alpha + \varphi_\alpha \leq 1$ and denoted by

$$\alpha = ((\varsigma_\alpha, \varphi_\alpha); r_\alpha).$$

C-IFVs are introduced as extensions of the IFVs in which each element is represented by a circle with the center of membership degree and non-membership degree (see Figure).

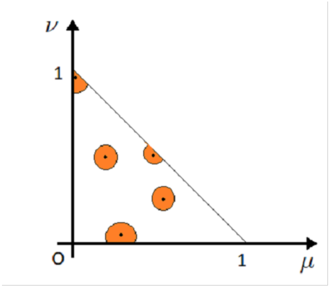


Figure: Geometric representation of C-IFVs

The concepts of t-norm and t-conorm are often used to define algebraic operations and aggregation operators for fuzzy sets. In addition, additive generators of t-norms and t-conorms play an important role while defining the algebraic operators.

Definition

The functions $T, S : [0, 1]^2 \rightarrow [0, 1]$ is called **a t-norm (or cornorm)** if it satisfy the Commutativity, Monotonicity, Associativity and identity properties.

Definition

A function $N : [0, 1] \rightarrow [0, 1]$ is called **a fuzzy negator** if for any $x, y \in [0, 1]$: N is continuous; $N(0) = 1$, $N(1) = 0$ and $N(x) \geq N(y)$ for $x \leq y$.

Definition

A decreasing function $g_t : [0,1] \to [0,\infty]$ with $g_t(1) = 0$ is called **the additive generator of** T given by $T(x,y) = g_t^{-1}(g_t(x) + g_t(y))$, while $h_t(t) = g_t(N(t))$ is **the additive generator of** S. A t-norm T is called **"Archimedean"** if $T(x,x) < x$ for any $x \in [0,1]$.

Definition

A (T,S,N) is said to be a dual triple, if $T(x,y) = N(S(N(x),N(y)))$ and $S(x,y) = N(T(N(x),N(y)))$ holds. Here, the term $S(x,y)$ dual t-conorm of T whenever $N(N(x)) = x$ for any $x \in [0,1]$."

The subtraction and division for C-IFVs can be proposed by using dual triples (T,S,N_I) where $N_I(x) = 1 - x$.

Atanassov 2020 has defined some algebraic operations for C-IFVs by considering the minimum t-norm and its dual t-conorm, maximum t-conorm, for the radius. Here instead of particular t-norms and t-conorms we use general ones to define algebraic operations and give some theoretical information.

Proposed operational laws

Definition

Let (T, S, N_I) be a dual triple and let R be a t-norm or a t-conorm. Then for C-IFVs $\alpha = ((\varsigma_a, \varphi_\alpha); r_\alpha)$, $\beta = ((\varsigma_\beta, \varphi_\beta); r_\beta)$ and $\lambda \geq 0$

i) $\alpha \oplus_R \beta = \left(\left(S\left(\varsigma_a, \varsigma_\beta\right), T\left(\varphi_a, \varphi_\beta\right)\right); R(r_\alpha, r_\beta)\right)$,

ii) $\alpha \otimes_R \beta = \left(\left(T\left(\varsigma_a, \varsigma_\beta\right), S\left(\varphi_a, \varphi_\beta\right)\right); R(r_\alpha, r_\beta)\right)$.

In addition; if T, S and R are Archimedean, g is "the additive generator" of T, $h_t(t) = g_t(N_I(t))$ and ρ is "the additive generator" of R, then

iii) $\lambda_R \alpha = \left(\left(h_t^{-1}\left(\lambda h\left(\varsigma_a\right)\right), g_t^{-1}\left(\lambda g\left(\varphi_a\right)\right)\right); \rho^{-1}\left(\lambda \rho\left(r_\alpha\right)\right)\right)$,

iv) $\alpha^{\lambda_R} = \left(\left(g_t^{-1}\left(\lambda g\left(\varsigma_a\right)\right), h_t^{-1}\left(\lambda h\left(\varphi_a\right)\right)\right); \rho^{-1}\left(\lambda \rho\left(r_\alpha\right)\right)\right)$.

Remark

If we take $g_t(t) = -\log t$, the "additive generator of the Algebraic t-norm" and $R(x, y) = \min/\max\{x, y\}$, then

i) $\alpha \oplus_{\min} \beta = \left(\left(\varsigma_a + \varsigma_\beta - \varsigma_a\varsigma_\beta, \varphi_a\varphi_\beta\right); \min\{r_\alpha, r_\beta\}\right)$,

ii) $\alpha \oplus_{\max} \beta = \left(\left(\varsigma_a + \varsigma_\beta - \varsigma_a\varsigma_\beta, \varphi_a\varphi_\beta\right); \max\{r_\alpha, r_\beta\}\right)$,

iii) $\alpha \otimes_{\min} \beta = \left(\left(\varsigma_a\varsigma_\beta, \varphi_a + \varphi_\beta - \varphi_a\varphi_\beta\right); \min\{r_\alpha, r_\beta\}\right)$,

iv) $\alpha \otimes_{\max} \beta = \left(\left(\varsigma_a\varsigma_\beta, \varphi_a + \varphi_\beta - \varphi_a\varphi_\beta\right); \max\{r_\alpha, r_\beta\}\right)$.

The inverses of operations $\oplus_{\min}$,$\oplus_{\max}$, $\otimes_{\min}$ and $\otimes_{\max}$ do not exist in general, but some certain circumstances. While defining the inverse operations, we use the function $C : \mathbb{R} \to [0,1]$ defined by

$$C(x) = \begin{cases} 0 & , \text{ if } x < 0 \\ x & , \text{ if } 0 \leq x \leq 1 \\ 1 & , \text{ if } x > 1. \end{cases}$$

Moreover, function C guaranties $r_{\alpha \oplus_R \beta} \in [0,1]$.

Definition

Let (T, S, N_I) be a dual triple and ρ be "the additive generator of Archimedean t-norm R". Then for C-IFVs $\alpha = ((\varsigma_a, \varphi_\alpha); r_\alpha), \beta = ((\varsigma_\beta, \varphi_\beta); r_\beta)$

i)

$$\alpha \ominus_R \beta = \begin{cases} \left(\begin{array}{c} \left(\begin{array}{c} h_t^{-1}\left(h\left(\varsigma_a\right) - h\left(\varsigma_\beta\right)\right), \\ g_t^{-1}\left(g\left(\varphi_a\right) - g\left(\varphi_\beta\right)\right) \end{array} \right); \\ C(\rho^{-1}\left(\rho\left(r_\alpha\right) - \rho\left(r_\beta\right)\right)) \end{array} \right) & , \text{ if } 0 \leq h\left(\varsigma_a\right) - h\left(\varsigma_\beta\right) \leq g\left(\varphi_a\right) - g\left(\varphi_\beta\right) \\ \left((0,1); C(\rho^{-1}\left(\rho\left(r_\alpha\right) - \rho\left(r_\beta\right)\right))\right) & , \text{ otherwise,} \end{cases}$$

ii)

$$\alpha \oslash_R \beta = \begin{cases} \left(\begin{array}{c} \left(\begin{array}{c} g_t^{-1}\left(g\left(\varsigma_a\right) - g\left(\varsigma_\beta\right)\right), \\ h_t^{-1}\left(h\left(\varphi_a\right) - h\left(\varphi_\beta\right)\right) \end{array} \right); \\ C(\rho^{-1}\left(\rho\left(r_\alpha\right) - \rho\left(r_\beta\right)\right)) \end{array} \right) & , \text{ if } 0 \leq h\left(\varphi_a\right) - h\left(\varphi_\beta\right) \leq g\left(\varsigma_a\right) - g\left(\varsigma_\beta\right) \\ \left((1,0); C(\rho^{-1}\left(\rho\left(r_\alpha\right) - \rho\left(r_\beta\right)\right))\right) & , \text{ otherwise.} \end{cases}$$

If we take $g_t(t) = -\log t$ in above definition, then we obtain

Remark

i) $$\alpha \ominus_R \beta = \begin{cases} \left(\left(\dfrac{\varsigma_a - \varsigma_\beta}{1-\varsigma_\beta}, \dfrac{\varphi_a}{\varphi_\beta} \right); \min\left\{1, \dfrac{r_\alpha}{r_\beta}\right\} \right) & , \quad \text{if } 1 \leq \dfrac{1-\varsigma_\beta}{1-\varsigma_\alpha} \leq \dfrac{\varphi_\beta}{\varphi_a} \\ \left((0,1); \min\left\{1, \dfrac{r_\alpha}{r_\beta}\right\} \right) & , \quad \text{otherwise,} \end{cases}$$

whenever $\rho(t) = -\log t$,

ii) $$\alpha \ominus_R \beta = \begin{cases} \left(\left(\dfrac{\varsigma_a - \varsigma_\beta}{1-\varsigma_\beta}, \dfrac{\varphi_a}{\varphi_\beta} \right); \max\left\{0, \dfrac{r_\alpha - r_\beta}{1-r_\alpha}\right\} \right) & , \quad \text{if } 1 \leq \dfrac{1-\varsigma_\beta}{1-\varsigma_\alpha} \leq \dfrac{\varphi_\beta}{\varphi_a} \\ \left((0,1); \max\left\{0, \dfrac{r_\alpha - r_\beta}{1-r_\alpha}\right\} \right) & , \quad \text{otherwise,} \end{cases}$$

whenever $\rho(t) = -\log(1-t)$,

iii) $$\alpha \oslash_R \beta = \begin{cases} \left(\left(\dfrac{\varsigma_a}{\varsigma_\beta}, \dfrac{\varphi_a - \varphi_\beta}{1-\varphi_\beta} \right); \min\left\{1, \dfrac{r_\alpha}{r_\beta}\right\} \right) & , \quad \text{if } 1 \leq \dfrac{1-\varphi_\beta}{1-\varphi_\alpha} \leq \dfrac{\varsigma_\beta}{\varsigma_a} \\ \left((1,0); \min\left\{1, \dfrac{r_\alpha}{r_\beta}\right\} \right) & , \quad \text{otherwise,} \end{cases}$$

whenever $\rho(t) = -\log t$,

iv) $$\alpha \oslash_R \beta = \begin{cases} \left(\dfrac{\varsigma_a}{\varsigma_\beta}, \dfrac{\varphi_a - \varphi_\beta}{1-\varphi_\beta}; \max\left\{0, \dfrac{r_\alpha - r_\beta}{1-r_\alpha}\right\} \right) & , \quad \text{if } 1 \leq \dfrac{1-\varphi_\beta}{1-\varphi_\alpha} \leq \dfrac{\varsigma_\beta}{\varsigma_a} \\ \left((1,0); \max\left\{0, \dfrac{r_\alpha - r_\beta}{1-r_\alpha}\right\} \right) & , \quad \text{otherwise} \end{cases}$$

whenever $\rho(t) = -\log(1-t)$.

Definition

For a collection of $\{\alpha_1, ..., \alpha_n\}$, a "weighted arithmetic and geometric" operator is defined as

$$WACIF_R(\alpha_1, ..., \alpha_n) := \omega_{1_R}\alpha_1 \oplus_R ... \oplus_R \omega_{n_R}\alpha_n =: (R)\bigoplus_{i=1}^{n} \omega_{i_R}\alpha_i \tag{1}$$

and

$$WGCIF_R(\alpha_1, ..., \alpha_n) := \alpha_1^{\omega_{1_R}} \otimes_R ... \otimes_R \alpha_n^{\omega_{n_R}} =: (R)\bigotimes_{i=1}^{n} \alpha_i^{\omega_{i_R}} \tag{2}$$

where $\omega_{i_R} \in [0, 1]$ is a weight vector with $\sum_{i=1}^{n} \omega_{i_R} = 1$.

Theorem

$WACIF(\alpha_1, ..., \alpha_n)$ and $WGCIF(\alpha_1, ..., \alpha_n)$ defined in above definition are C-IFVs where

$$WACIF_R(\alpha_1, ..., \alpha_n) = \begin{pmatrix} \left(h_t^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} h_t(\varsigma_{\alpha_i})\right), g_t^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} g_t(\varphi_{\alpha_i})\right)\right); \\ \rho^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} \rho(r_{\alpha_i})\right) \end{pmatrix} \quad (3)$$

and

$$WGCIF_R(\alpha_1, ..., \alpha_n) = \begin{pmatrix} \left(g_t^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} g_t(\varsigma_{\alpha_i})\right), h_t^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} h_t(\varphi_{\alpha_i})\right)\right); \\ \rho^{-1}\left(\sum_{i=1}^{n} \omega_{i_R} \rho(r_{\alpha_i})\right) \end{pmatrix}. \quad (4)$$

Remark

i) *If* $g_t(t) = -\log t$, $h_t(t) = -\log(1-t)$ *and* $\rho(t) = -\log t$ *in above definition, then we have*

$$WA_ICIF(\alpha_1, ..., \alpha_n) = \left(\left(1 - \prod_{i=1}^{n}\left(1 - \varsigma_{\alpha_i}\right)^{\omega_{i_R}}, \prod_{i=1}^{n}\varphi_{\alpha_i}^{\omega_{i_R}}\right); \prod_{i=1}^{n} r_{\alpha_i}^{\omega_{i_R}}\right)$$

and

$$WG_ICIF(\alpha_1, ..., \alpha_n) = \left(\left(\prod_{i=1}^{n}\varsigma_{\alpha_i}^{\omega_{i_R}}, 1 - \prod_{i=1}^{n}\left(1 - \varphi_{\alpha_i}\right)^{\omega_{i_R}}\right); \prod_{i=1}^{n} r_{\alpha_i}^{\omega_{i_R}}\right)$$

which are called the "Type I Algebraic weighted arithmetic and geometric aggregation operators", respectively.

ii) *If* $g_t(t) = -\log t$, $h_t(t) = -\log(1-t)$ *and* $\rho(t) = -\log(1-t)$ *in above definition, then we have*

$$WA_{II}CIF(\alpha_1, ..., \alpha_n) = \left(\left(1 - \prod_{i=1}^{n}\left(1 - \varsigma_{\alpha_i}\right)^{\omega_{i_R}}, \prod_{i=1}^{n}\varphi_{\alpha_i}^{\omega_{i_R}}\right); 1 - \prod_{i=1}^{n}\left(1 - r_{\alpha_i}\right)^{\omega_{i_R}}\right)$$

and

$$WG_{II}CIF(\alpha_1, ..., \alpha_n) = \left(\left(\prod_{i=1}^{n}\varsigma_{\alpha_i}^{\omega_{i_R}}, 1 - \prod_{i=1}^{n}\left(1 - \varphi_{\alpha_i}\right)^{\omega_{i_R}}\right); 1 - \prod_{i=1}^{n}\left(1 - r_{\alpha_i}\right)^{\omega_{i_R}}\right)$$

which are called the "Type II Algebraic weighted arithmetic and geometric aggregation operators", respectively.

In this section, we provide an extended EDAS method with C-IFS environment by using the subtraction and division operations. We assume that one of R and Q is an Archimedean t-norm and other one is the dual t-conorm of this t-norm.

- **Step 1**: Construct a MCDM problem with m alternatives $\{\mathcal{A}_1, \mathcal{A}_2 ..., \mathcal{A}_m\}$ and n criteria $\{x_1, ..., x_n\}$.
- **Step 2**: Evaluate each alternative under each criteria and recorded their information in terms of C-IFV and hence summarized in a decision matrix $\mathcal{L}$ as

$$\mathcal{L} = \begin{bmatrix} l_{11} & \cdots & l_{1n} \\ \vdots & \ddots & \vdots \\ l_{m1} & \cdots & l_{mn} \end{bmatrix}$$

  where l_{ij} is a C-IFV for each $i = 1, ..., m$ and $j = 1, ..., n$.

- **Step 3**: Obtain an average solution by

$$AV := \left[AV_j\right]_{1\times n}$$

where

$$AV_j = WACIF_R(l_{1j}, ..., l_{nj}) \in C - IFV(X)$$

or

$$AV_j = WGCIF_R(l_{1j}, ..., l_{nj}) \in C - IFV(X)$$

with the weights $\omega_{i_R} = \dfrac{1}{m}$ for any $i = 1, ..., m$.

- **Step 4**: Evaluate the positive distance $d^+ = \left[d^+_{ij}\right]$ and the negative distance $d^- = \left[d^-_{ij}\right]$ from the average solution where

$$d^+_{ij} = \left(l_{ij} \ominus_R AV_j\right) \oslash_Q AV_j,$$

$$d^-_{ij} = \left(AV_j \ominus_R l_{ij}\right) \oslash_Q AV_j$$

whenever x_j is the "benefit criteria" and whenever x_j is the "cost criteria"

$$d^+_{ij} = \left(AV_j \ominus_R l_{ij}\right) \oslash_Q AV_j,$$

$$d^-_{ij} = \left(l_{ij} \ominus_R AV_j\right) \oslash_Q AV_j.$$

- **Step 5:** Calculate the weighted sums for wd_i^+ and wd_i^- by

$$wd_i^+ = WACIF_R(d_{i1}^+, ..., d_{im}^+) \quad ; wd_i^+ = WGCIF_R(d_{i1}^+, ..., d_{im}^+)$$

and

$$wd_i^- = WACIF_R(d_{i1}^-, ..., d_{im}^-) \quad ; wd_i^- = WGCIF_R(d_{i1}^-, ..., d_{im}^-)$$

with $\omega_{i_R} \in [0, 1]$ is a weight vector and $\sum_{i=1}^{n} \omega_{i_R} = 1$.

- **Step 6:** Defuzzify wd_i^+ and wd_i^- for each $i = 1, ..., m$ by using the relative score function (RSF) of Kahraman 2022 and obtain

$$Dwd_i^+ = \left(\frac{\frac{1}{r_{wd_i^+}}}{\left(\sum_{i=1}^{m} \frac{1}{r_{wd_i^+}^2} \right)^{1/2}} \right)^{0.01} \frac{\left(1 - \varphi_{wd_i^+}\right)\left(1 + \varsigma_{wd_i^+}\right)}{3} \in [0, 1]$$

and

$$Dwd_i^- = 1 - \left(\frac{\frac{1}{r_{wd_i^-}}}{\left(\sum_{i=1}^{m} \frac{1}{r_{wd_i^-}^2} \right)^{1/2}} \right)^{0.01} \frac{\left(1 - \varphi_{wd_i^-}\right)\left(1 + \varsigma_{wd_i^-}\right)}{3} \in [0, 1].$$

- **Step 7**: Evaluate the appraisal score (AS) for each alternative $i = 1, ..., m$ by

$$AS_i = \frac{Dwd_i^+ + Dwd_i^-}{2}$$

and "rank the alternatives" with respect to AS values. The alternative with the "highest AS_i" value is "the best alternative".

Remark

Contrary to classical EDAS method normalization step of weighted sums for d^+ and d^- is not needed in the extended EDAS method since wd_i^+, wd_i^- are still C-IFVs naturally.

The steps of the proposed extended EDAS method is visualized in Figure below:

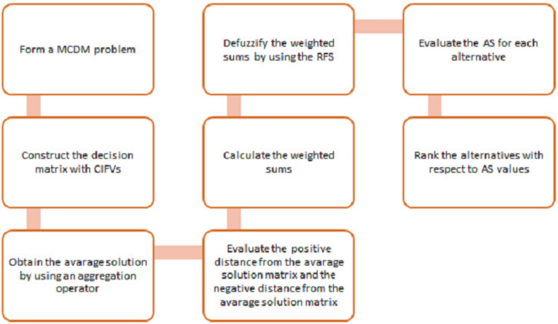


Figure: Flowchart of the proposed extended EDAS method

To illustrate the stated method, we take an example from the literature Kahraman 2022. This is an MCDM problem with three alternatives $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ and three criteria $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$. The decision matrix related to this problem is listed in Table Kahraman 2022 below:

Table: Input data related to the given alternatives

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
$\mathcal{A}_1$	$((0.381, 0.567);0.308)$	$((0.412, 0.497);0.081)$	$((0.566, 0.359);0.122)$
$\mathcal{A}_2$	$((0.466, 0.446);0.269)$	$((0.443, 0.477);0.154)$	$((0.649, 0.267);0.129)$
$\mathcal{A}_3$	$((0.316, 0.536);0.203)$	$((0.383, 0.539);0.252)$	$((0.494, 0.442);0.143)$

So, Steps 1 and 2 are applied.

- **Step 3:** By using WA_ICIF operator, given in (i) of above remark, we compute the average solution and the result is obtained as

$$AV = \left\{ \begin{array}{c} (C_1, ((0.3908, 0.5137)\,; 0.2562))\,, (C_2, ((0.4097, 0.5037)\,; 0.1465))\,, \\ (C_3, ((0, 5744, 0.3486)\,; 0.1310)) \end{array} \right\}$$

However, by using other proposed operators, the average solutions are listed in Table below:

Table: The average solutions

	C_1	C_2	C_3
AV (WA_ICIF)	$((0.3908, 0.5137)\,; 0.2562)$	$((0.4097, 0.5037)\,; 0.1465)$	$((0, 5744, 0.3486)\,; 0.1310)$
AV (WG_ICIF)	$((0.3828, 0.5190)\,; 0.2562)$	$((0.4088, 0.5050)\,; 0.1465)$	$((0.5661, 0.3600)\,; 0.1310)$
AV ($WA_{II}CIF$)	$((0.3908, 0.5137)\,; 0.2613)$	$((0.4097, 0.5037)\,; 0.1653)$	$((0.5744, 0.3486)\,; 0.1314)$
AV ($WG_{II}CIF$)	$((0.3828, 0.5190)\,; 0.2613)$	$((0.4088, 0.5050)\,; 0.1653)$	$((0.5661, 0.3600)\,; 0.1314).$

- **Step 4:** Compute the d^{+} and d^{-} from average solution and results are listed in First and second Tables, respectively.

Table: Positive distance from the average solution matrix with respect to WA_ICIF

	C_1	C_2	C_3
$\mathcal{A}_1$	$((0,1);1)$	$((0.0096,0.9733);0.4762)$	$((0,1);0.9206)$
$\mathcal{A}_2$	$((0.3159,0.7291);1)$	$((0.0964,0.8933);1)$	$((0.3051,0.6406);0.9820)$
$\mathcal{A}_3$	$((0,1);0.7208)$	$((0,1);1)$	$((0,1);1)$

Table: Negative distance from the average solution matrix with respect to WA_ICIF

	C_1	C_2	C_3
$\mathcal{A}_1$	$((0.0405,0.8066);0.7739)$	$((0,1);1)$	$((0.0338,0.9556);1)$
$\mathcal{A}_2$	$((0,1);0.9361)$	$((0,1);0.9428)$	$((0,1);1)$
$\mathcal{A}_3$	$((0.2798,0.9144);1)$	$((0.1056,0.868);0.5094)$	$((0.2767,0.6756);0.9038)$

- **Step 5:** Taking the equal weights $\omega_i = \dfrac{1}{3}$ for the criteria, the computed values of the weighted sums wd_i^+ and wd_i^- are given as

$$wd_1^+ = ((0.0032, 0.9910)\,; 0.7597) \quad ; \quad wd_1^- = ((0.0249, 0.9169)\,; 0.9181)$$
$$wd_2^+ = ((0.2455, 0.7472)\,; 0.9940) \quad ; \quad wd_2^- = ((0, 1)\,; 0.9592)$$
$$wd_3^+ = ((0, 1)\,; 0.8966) \quad ; \quad wd_3^- = ((0.2248, 0.8124)\,; 0.7722)$$

- **Step 6:** The defuzzified values wd_i^+ and wd_i^- are obtained as

$$Dwd_1^+ = 0.003 \quad ; \quad Dwd_2^+ = 0.1043 \quad ; \quad Dwd_3^+ = 0$$
$$Dwd_1^- = 0.0282 \quad ; \quad Dwd_2^- = 0 \quad ; \quad Dwd_3^- = 0.0763$$

- **Step 7:** The appraisal score AS_i is evaluated as $AS_1 = 0.4874$ $AS_2 = \mathbf{0.5522}$ and $AS_3 = 0.4619$. Hence, we get the ranking $A_2 \succ A_1 \succ A_3$ with respect to weighted aggregation operator WA_ICIF.

Comparison with the Remaining A.O.

When we also solve the same MCDM problem with the aggregation operators WG_ICIF, $WA_{II}CIF$ and $WG_{II}CIF$, we obtain exactly the same ranking $A_2 \succ A_1 \succ A_3$. Note also that these results are in agreement with the result of Kahraman 2022.

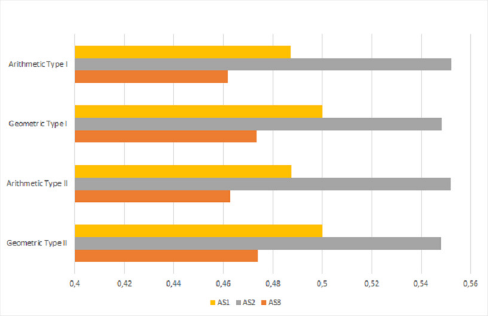


Figure: Comparison of the rankings with respect to all weighted aggregation operators

In this sub-section, we compare the proposed extended EDAS method with several current decision making techniques to evaluate its reliability and effectiveness:

Table: Comparison of result of existing methods

Existing Methods	Scores			Ranking	Best Selection
	A_1	A_2	A_3		
VIKOR (pessimistic) method by **Kahraman and Otay,2022**	0.7050	**0.0000**	0.9650	$A_2 > A_1 > A_3$	A_2
VIKOR (optimistic) method by **Kahraman and Otay,2022**	0.2940	1.000	**0.0000**	$A_3 > A_1 > A_2$	A_3
TOPSIS method by **Alkan and Kahraman,2022**	0.5178	**0.6146**	0.3469	$A_2 > A_1 > A_3$	A_2
AHP-VIKOR method by **Otay and Kahraman,2022**	0.6720	**0.0000**	1.0000	$A_2 > A_1 > A_3$	A_2

Now, we evaluate the ranking consistency of the MCDM problem by using Eq. (5) of the “Spearman’s Rank Correlation Coefficient” and the test results are shown in Table below:

$$\rho =: 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} d_i^2, \qquad (5)$$

where n is the number of results, and d_i denotes the difference in the results’ ranks.

Table: The Spearman's rank correlation coefficient for MCDM problem

Method	$WA_{I}CIF$	$WG_{I}CIF$	$WA_{II}CIF$	$WG_{II}CIF$
VIKOR (pessimistic) method by **Kahraman and Otay,2022**	1	1	1	1
VIKOR (optimistic) method by **Kahraman and Otay,2022**	-1	-1	-1	-1
TOPSIS method by **Alkan and Kahraman,2022**	1	1	1	1
AHP-VIKOR method by **Otay and Kahraman,2022**	1	1	1	1

The obtained values are considered as highly valid range because they are greater than 0.71 **Spearman,1987** except for the result of **Kahraman and Otay,2022**'s method based on VIKOR (optimistic). According to the above Table, it is seen that the results obtained with the pessimistic perspective or the combination of pessimistic and optimistic perspectives as in **Otay and Kahraman,2022** are compatible with the results obtained in this paper, but the results obtained by solving with the optimistic perspective are both with the results obtained by the pessimistic method and the results obtained in this paper indicates inconsistency.


THANK
YOU!