***A*MTLR**: *A graduate school for **A**dvancing research on the **M**athematics **T**eaching-**L**earning **R**elationship*

OBJECTIVE

A crucial question in educational systems all over the world is how to better support teachers in supporting students' learning of mathematics. For this purpose, substantial efforts have been put into curriculum reforms and large-scale professional development programmes. While some impact on teacher practices has been observed (e.g., Österholm et al. 2016) it has been repeatedly shown that obtaining substantial positive effects on students' learning through such efforts is difficult (Gravemeijer et al. 2016; Lindvall et al. 2022, Ryve & Cobb 2024), and that ineffective mathematics teaching practices, such as overuse of routine tasks (Boesen et al. 2014) and procedural feedback (Stovner & Klette 2022), are still common.

Based on the existing research on professional development, Sims et al. (2023) suggest that bringing about changes in teachers' practice require the combination of four elements: *insight* about teaching and learning, *motivation* for making changes, *techniques* for putting the insights to work, and support for *embedding* the techniques in teachers' practice. For such changes to improve students' learning, one element is even more fundamental: that the insights are based on a robust understanding of *which* properties of mathematics teaching that lead to better mathematics learning, as well as *how* and *why* (Sims et al. 2023, Palm et al. 2023). To give rise to techniques, this understanding needs to be specific enough to provide guidance for teachers' instructional decisions (diSessa & Cobb 2003). To support embedding in practice, this understanding needs to be general enough to be applicable in a substantial subset of situations in a variety of contexts for a variety of students (Lowrie 2024).

Mathematics education research has long acknowledged that while we have made progress in designing and implementing mathematics teaching that seems to promote students' learning, we face challenges in documenting that improved learning outcomes are results of teaching rather than other variables (Niss 2007). In particular, a shortfall of interventionist and experimental studies directly investigating the mathematics teaching-learning relationship has been consistently noticed (Alcock et al. 2013, Inglis & Foster 2018, Stylianides & Stylianides 2013). As a result, some central claims about the mathematics teaching-learning relationship that are made and repeated go beyond existing empirical warrants (Otten et al. 2017).

If we are to improve future mathematics teaching and learning, there is an urgent need to develop the competence of future mathematics education researchers in methods for investigating the mathematics teaching-learning relationship, and the competence of future mathematics teacher educators in evaluating evidence for claims about this relationship. **The objective** of the graduate school AMTLR is to advance research on the mathematics teaching-learning relationship. More specifically, the doctoral students will develop the competence to design, conduct and critically evaluate research that contribute to answering the fundamental question: *How and why do specific mathematics teaching designs lead to specific mathematics learning outcomes?*

The expected results of the graduate school are threefold: (1) new and more robust results regarding the mathematics teaching-learning relationship, which can be incorporated in teacher education and teacher professional development, strengthening their research base, (2) methodological advancements for investigating the mathematics teaching-learning relationship, and (3) doctors that are competent in how to (a) overview and evaluate existing support, and (b) build new support for how mathematics teaching should be designed to promote students' learning, and can utilise this competence to increase the quality of mathematics education research and mathematics teacher education. To achieve these results, AMTLR entails five specifically developed courses (45 ETCS in total), regular work-in-progress seminars, and support from strong research environments where the doctoral students are connected to established research groups who offer apprenticeship within their long-term research programmes. AMTLR is organised by a coordinator and a board of representatives from each partner university, gathering expertise in the focus of AMTLR and broad experience in doctoral education, and is supported by an international advisory board.

State-of-the-art research regarding the mathematics teaching-learning relationship

In this section, we explain why we currently do not have sufficient robust knowledge regarding the mathematics teaching-learning relationship. Questions regarding this relationship are causal questions: we teach to cause learning. To answer such questions and, particularly, questions regarding what specific teaching designs lead to specific learning outcomes, *experimental approaches* are necessary, that is, approaches entailing comparisons of effects of different teaching interventions on students' mathematics learning. While we use this term inclusively to embrace a variety of methods—e.g., randomised controlled trials, quasi-experiments, educational design research and teaching experiments—we are here specifically interested in their use for investigating aspects of the mathematics teaching-learning relationship relevant for teacher education. Other types of studies, such as investigations of students' difficulties and learning paths, can undoubtedly contribute to the formulation of hypotheses regarding the mathematics teaching-learning relationship, but they are insufficient for producing results regarding it.

Experimental approaches seem to grow in and out of favour in educational research, which was noticed early on (Campbell & Stanley 1963). In their review of the two most prominent mathematics education (ME) research journals, Inglis and Foster (2018) identified a shift away from experimental approaches from 1970 to 2015, so dramatic that they call it an "experimental cliff". Our analysis of the 291 articles published 2024–2025 in the four most prominent journals focusing on teaching and learning in mathematics reveals that experimental approaches are still scarce: only 22 of the articles (<8 %) report studies that compare effects on learning from different interventions in pre-school and school mathematics teaching.

Although Inglis and Foster (2018) found that experimental approaches are rare in ME journals, they also found that such approaches are still used for investigating the mathematics teaching-learning relationship. However, they are reported in educational psychology and cognitive (EPC) research journals. Though there are examples of collaborations across ME and EPC research (e.g., in one of our research groups, Lithner 2017, Stillesjö et al. 2021), the current state can be characterised as a split in two strands (Resnick & Steiff 2024). Regarding some issues, these two strands produce conflicting results. For example, EPC researchers have, for at least 60 years, consistently stated that they have sufficient research evidence to claim that problem-based and inquiry teaching designs are less effective than direct instruction (Ausubel 1961, Kirschner et al. 2006, Sweller et al. 2024), while ME researchers claim that they have sufficient research evidence for the opposite (Hiebert et al. 2025, Lester & Cai 2016). Regarding other issues, the lack of communication between the two strands hinders synthesis of results. As an example, the EPC research on explanatory questioning—asking learners to provide explanations and arguments—has been described as lacking studies situated in mathematical classrooms (Star & Verschaffel 2017), though there is extensive ME research investigating such questioning used by teachers in classroom interactions (e.g., Franke et al. 2009, Ellis et al. 2019). Evidently, there is a lack of consistent and synthesised understanding of the mathematics teaching-learning relationship that can constitute the insight on which to build successful pre- and in-service teacher education.

The difficulties in bridging the two strands stem from both theoretical and methodological differences. The theoretical differences concern both *what* phenomena key theories focus on, with EPC research focusing cognitive aspects of learning and ME research increasingly focusing on social aspects (Inglis & Foster 2018, Resnick & Steiff 2024), and *how* theories are worked with, where EPC research centres testing of and accumulating support for theories, while replication and even revisiting proposed theories is rare in ME research (Lowrie 2024). In addition, EPC research aims for general descriptive and explanatory theory for learning processes, while ME research has moved towards more contextually specific results (Resnick & Steiff 2024). Some educational researchers even suggest abandoning claims of generality in favour of rich descriptions of contexts (Hirsch et al. 2022). While the increased acknowledgement that generality of results is not easily achieved is an important development in educational research, it seems evident that when the aim is insight that can form the basis of successful pre- and in-service teacher education, it is rather necessary to increase our efforts to reach generality of results; if we cannot say anything about whether a teaching design would work for other teachers, it seems unwarranted to disseminate it.

Methodologically, the difference mainly concerns the use of experimental methods, as described above. EPC research is characterised by experimental methods in laboratory settings, while mathematics education research is characterised by situated, descriptive, observational methods in real classrooms (Resnick & Steiff 2024). A less discussed difference—that is nonetheless readily noted when reviewing research in both fields—is that EPC studies often measure learning outcomes immediately after a short session of teaching, while ME studies seemingly always measure learning outcomes after longer delays. In relation to causal claims, one may describe it as EPC studies having greater internal validity, due to stricter and more controlled research designs, and ME studies having greater external validity, due to them being more similar to teachers' practice.

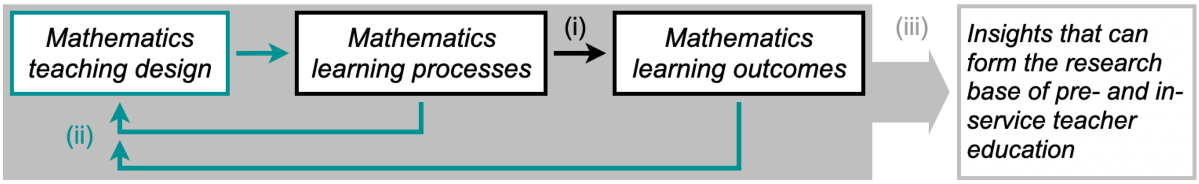
The differences could be advantageous if the two strands informed each other to leverage their relative strengths and weaknesses, but as described above, this is seldom the case. This does not only hinder advancements in research, but also improvement of mathematics teaching and teacher education, since it makes it difficult to draw conclusions about what, how and why teaching designs support student learning. To advance the study of the mathematics teaching-learning relationship, the doctoral programme AMTLR aims to build on and learn from the state-of-the-art in both these strands, theoretically as well as methodologically.

Focus of the graduate school AMTLR

The causal relationship between teaching and learning is an inherently complex phenomenon that is not readily predicted, explained, and reproduced. To support the endeavour of studying this relationship, the educational activities and individual doctoral projects within AMTLR will (a) be guided by a model that clarifies the key focused sub-relationships, and (b) centre five methodological components that are crucial for developing new and robust knowledge about the mathematics teaching-learning relationship. We will present (a) and (b) in the next two sections.

Focused sub-relationships within AMTLR

AMTLR focuses on the elements and relationships foregrounded in the following model:



The rationale for this model is the following: (i) *Teaching* (T) does not directly cause *learning outcomes* (LO) but indirectly through engaging students in *learning processes* (LP) that in turn result in new knowledge and competence: T→LP→LO (Hiebert et al. 2025, Palm et al. 2023). (ii) The same teaching design will not lead to the same learning for all students in all settings at all times. Teaching and learning do not take place in a vacuum but interacts with cognitive and contextual factors (Palm et al. 2023, Resnick & Steiff 2024). This does not mean that teachers need to act totally different in every situation. Rather, it means that to effectively support students' learning, teachers need to continuously assess individual students' knowledge and behaviour in relation to desired learning processes and outcomes and provide accordingly adapted feedback and instructional activities. These components are at the heart of several teaching practices, such as formative assessment, noticing and responsive teaching, whose positive effects on student learning are well-documented (e.g., Lee et al. 2020). A key property of teaching design is thus how observed learning processes and learning outcomes feed back into adaptations of teaching: T←LP and T←LO.

(iii) If the aim is to produce insights that are useful for teachers (Sims et al. 2023), this aim needs to be factored in from the start. The insights have to be both (a) specific enough to provide guidance for teachers' instructional decisions on a level that is consequential for students' learning (diSessa & Cobb 2003), and (b) general enough to be applicable in a substantial subset of situations in a variety of contexts for a variety of students (Lowrie 2024). (a) implies that the constructs used to describe the key elements (teaching, learning processes and learning outcomes) need to be defined in terms of properties that are accessible to and meaningful for teachers, for example observable indicators for key learning processes and ways of measuring learning outcomes aligned with teachers' goals. (b) implies that claims about the mathematics teaching-learning relationship need sufficient range and external validity, requiring testing in multiple contexts (Resnick & Steiff 2024).

Focused methodological components within AMTLR
To reach the aims of AMTLR, the individual projects will all—although to various extent and in various ways—include five methodological components: (1) **Testing specific hypotheses about the mathematics teaching-learning relationship**: Naturally, each project needs to specify the model above in relation to more specific research questions, which we cannot yet know. However, to formulate interesting and yet undecided hypotheses, the doctoral students need both broad and in-depth knowledge of existing theories about the mathematics teaching-learning relationship from both ME and EPC research, including their underlying arguments and empirical backing. (2) **Comparisons**: Comparisons are key for obtaining results about how teaching affect learning, but they can be made in different ways, for example, between different groups subject to different teaching or between results of different teaching for the same student or group. Analyses may include both qualitative approaches and statistical tests, depending on research questions. (3) **Interventions**: Deliberately manipulating the independent variable—in our case, teaching—is generally seen as a key feature in studying causal relationships. Therefore, most projects may entail the design and use of interventions. Interventions can be conducted in classroom settings or with individual or small groups of students in laboratory settings, as individual studies or in the context of an iterative design research approach (Nilsson & Eckert 2024, Säfström & Lithner 2025). (4) **Measuring learning outcomes**: Since the focus is on the mathematics teaching-learning relationship, it is of interest to measure learning outcomes after teaching. This can be done using written mathematical tests, but also by other methods, such as interviews or observing student participation in subsequent learning activities. (5) **Observation of learning processes**: If one aims to explain *how* teaching affects learning, including observations of the actual learning processes is often superior to merely measuring outcomes. Such observation can be conducted, e.g., by using field notes, audio and/or video recordings, collection of artefacts, think-aloud-protocols or eye-tracking equipment. All projects might not include both (4) and (5), but at least of them is necessary to investigate the mathematics teaching-learning relationship.

The focal point of individual projects may be the teaching and learning of particular mathematical topics, such as early algebra or probability reasoning at upper secondary school, or the design and function of key elements of mathematics teaching, such as task sequences or teacher support during problem solving. In addition, as the projects aim to contribute to professional insight, motivation for change, teaching techniques and embedding in practice (Sims et al. 2023), the projects will likely also result in new tools for observing and assessing mathematics learning processes and outcomes that are accessible and functional for teachers.

RESEARCH ENVIRONMENT

One reason for the lack of research directly studying the mathematics teaching-learning relationship is that it is inherently difficult and requires extensive effort. For doctoral students to take on this endeavour, they need learning opportunities to develop the necessary competence, not only in terms of courses, but also as apprenticeship in research environments where senior researchers run several projects at various stages of development (Herbst 2025) in established teacher-researcher partnerships (Prediger et al. 2024). The three partners of AMTLR offer such research environments to our doctoral students:

Umeå Mathematics Education Research Centre (UMERC) at **Umeå University (UMU)** has been one of the largest mathematics education research groups in the Nordic region for 20 years. It has about 30 members, including four full professors. The last 5 years, members of UMERC have received external funding for 14 different projects from the Swedish Research Council (SRC), Swedish Institute for Educational Research (SIER), and Wallenberg foundations, with 11 as PI, and a total budget of 67 MSEK. UMERC has successfully collaborated in national research schools in mathematics education, has ongoing international research collaborations with researchers from eight countries, and extensive experience of research collaboration with teachers, schools, municipalities and national educational agencies (e.g., the National Agency for Education and the National Schools Inspectorate). The research within UMERC is organised in three research groups, that all study particular aspects of the mathematics teaching-learning relationship using a variety of research methods, such as long-term research and development projects based on collaborations with teachers (Boström & Palm 2023), design research methodology (Säfström et al. 2024, Säfström & Lithner 2025), RCTs (Andersson & Palm 2017), eye-tracking (Norqvist et al. 2023) and brain imaging (Stillesjö et al. 2021).

The mathematics education research group at **Linnaeus University (LNU)** consists of three full professors, one associate professor, and six assistant professors. Their research is united around practice-based intervention research in collaboration with mathematics teachers at all educational stages, and focuses on communication and learning in mathematics, mathematics teachers' identity and professional development, younger children's learning in mathematics, and digitally enhanced mathematics teaching. The last 10 years, the group have received over 30 MSEK in external funding, including three projects from SIER and one from SRC.

The ME research group at **Mälardalen University (MDU)** comprises two full professors, assistant professors, and several other researchers and educators engaged in various research projects. Their primary focus is on enhancing classroom practices through research in professional development, practice-based mathematics teacher education, and curriculum resources. The last 10 years, the group has secured substantial external funding amounting to 96 MSEK for these initiatives, including six projects funded by SRC and numerous large-scale collaborative projects together with municipalities. In these projects, the importance of theories and methodologies for understanding the relation between students' mathematical learning and effective teaching methods is typically a central focus.

PROGRAMME DESCRIPTION

The projects in AMTLR are to investigate and theorise the mathematics teaching-learning relationship. The doctoral students are to become both proficient researchers in mathematics education and teacher educators knowledgeable in how the research base for claims about the mathematics teaching-learning relationship can be evaluated. This means that AMTLR intends not only to educate proficient researchers in mathematics education in general, but also to educate them in the specific methods and approaches needed to investigate, interpret and evaluate the mathematics teaching-learning relationship. To achieve this, AMTLR includes the following educational activities: (1) **Five compulsory courses of a total of 45 ECTS** that will both orient the doctoral students in the wider fields of ME and EPC research, and develop specific competence regarding theoretical and methodological aspects of research concerning the mathematics teaching-learning relationship. (2) **Yearly work-in-progress seminars** (WPSs) where all AMTLR students meet and present their ongoing work and current issues amongst them and with senior researchers. (3) **High-quality supervision and support from strong research groups** that conduct research on the mathematics teaching-learning relationship. A substantial part of the supervision will be provided by the members of the board, and additional supervisors will be appointed to assure the right competence for each student's project. (4) **Two teacher educator conferences** (TECs) where AMTLR students build experience in disseminating research results and formulating implications for teacher education. For details about courses, WPSs and TECs, see Organisation.

To support national and international networking and to further the doctoral students' skills in presenting and discussing their scholarly work orally and in writing, AMTLR will provide funding for and work actively to support the participation in at least two national or international conferences for each doctoral student. In addition, each university provides additional courses focusing generic skills, such as research ethics, academic writing, and introduction to educational research, as well as specific qualitative and quantitative methods of analysis. The research environments at each university also provide seminar series where doctoral students can learn from, present for, and receive feedback from other researchers.

The structure and format of AMTLR is developed to suit the objective of advancing research on the mathematics teaching-learning relationship. In particular, two aspects are key: (1) AMTLR is exclusively in mathematics. This allows us to develop courses which directly address the concerns and questions related to mathematics teaching and learning and are able to cover perspectives from both ME and EPC research. (2) The three participating universities all have established research groups that investigate the mathematics teaching-learning relationship, have extensive experience from PhD training and collaboration in graduate schools, and extensive networks including international scholars in the field as well as practising teachers, schools and school administrators. This allows the students to draw on comprehensive research experience and scientific competence within the theme of the graduate school.

The recruitment base for AMTLR consist of people with either a teacher or a mathematics degree, most likely working as school teachers or teacher educators, who are interested in pursuing a career in mathematics teaching education research and mathematics teacher education. The collaborating universities announce such doctoral student positions regularly and have 5–10 qualified applicants per position. We expect similar numbers of applicants or higher to AMTLR, since doctoral degrees is now part of the national merit system for teachers.

ORGANISATION

AMTLR will employ at least 9 doctoral students, with at least 3 placed at each university. The doctoral students are enrolled at the university where they are placed, and each university is fully responsible for their students' examination, supervision, and follow-up of the individual study plans, which will be handled in line with local regulations at each university. Each of the participating universities is responsible for planning and teaching 15 ECTS, based on consultations with the AMTLR board (see below).

Courses: AMTLR includes five compulsory courses of a total of 45 ECTS. The responsibility of each course is placed at one of the partner universities, with guest lectures from and cross-over activities between the partner universities when suitable. The courses are taught as a combination of sessions held at the university responsible for the course (2x2 days/7.5 ECTS) and online supervision and communication between participants.

Course 1. *Introducing research on the mathematics teaching-learning relationship* (7.5 ECTS, UMU). This course will orient the doctoral students in the broader field of mathematics education research historically and currently—its questions, methods, and results—as well as EPC research on mathematics teaching and learning. It will situate research on the mathematics teaching-learning relationship within this field and develop knowledge of the particular advancements and challenges regarding research with this focus.

Course 2. *Connecting theories of mathematics teaching and learning* (15 ECTS, MDU). This course delves into the conceptualizations and theories related to mathematics learning and teaching across multiple disciplines, including EPC and ME research, and examines their origins, assumptions, developments, phenomena, and applications. By utilizing examples of specific theories as well as frameworks of types of theories, the course aims to elucidate the state-of-the-art research regarding the mathematics teaching-learning relationship.

Course 3. *Research design for studying the mathematics teaching-learning relationship* (7.5 ECTS, UMU). This course will develop knowledge of and ability to choose, adapt and critically evaluate research designs for investigating the teaching-learning relationship. The course will cover key experimental and quasi-experimental designs for generalised causal inference in the social sciences (Shadish et al. 2002), research designs developed within education and ME research, such as microgenetic research methodology (Simon 2018) and design research (Cobb et al. 2003, Säfström & Lithner 2025), and how multiple studies can be employed to contribute to building robust research bases for causal claims.

Course 4. *Working with data in mathematics education research* (7.5 ECTS, LNU). This course equips students with theoretical and practical tools to generate, organize, and analyse data for exploring the mathematics teaching-learning relationship from both ME and EPC research. The course guides students through both qualitative and quantitative research strategies, emphasizing their respective affordances and limitations. Students will explore qualitative methods such as open coding, thematic coding, and discourse analysis alongside statistical techniques, survey analysis, and measurement frameworks. Attention is given to mixed-methods approaches, including how to design studies that meaningfully integrate diverse data sources for investigating the mathematics teaching-learning relationship.

Course 5. *Intervention research in mathematics education* (7.5 ECTS, LNU). This course explores how interventions can be developed and used for testing specific hypotheses about the mathematics teaching-learning relationship. The course encompasses the relationships between data, context, interventions, methods for evaluating the impact of interventions on learning, the different roles between researchers and practitioners in design and theory building, and ethical considerations regarding interventions in mathematics education.

Work-in-progress seminars (WPSs): Besides the common courses, the doctoral students shall participate in a series of common *Work in Progress Seminars* (WPS). During a WPS, the students discuss each other's work and provide feedback and progress reviews, with senior researchers acting as discussants. The themes for the WPSs will be finally decided by the board but are suggested to be: WPS1: project plans, aims and research questions in relation to existing research and theories, WPS2: study design and methods for data collection and analysis, WPS3: analyses, preliminary results, and further research, WPS4: synthesis of sub-studies and empirical, methodological and theoretical contribution of the thesis, WPS5: dissemination of results and post-dissertation plans. The location of the WPSs will alternate between the partner universities and the coordinator and the university representative will be responsible for their organisation.

Teacher educator conferences (TECs): AMTLR will arrange two online conferences where the doctoral students present their work and findings for teacher educators, one mid-conference focusing methods and preliminary results and one post-conference focusing results and need for further research. To enable teacher educators at all three universities to participate, these conferences will be held online, and invitations will be extended to other universities.

The activities will take place as follows: Year 1: Course 1–2, WPS1; Year 2: Course 3–5, WPS2; Year 3: WPS3, TEC1; Year 4: WPS4; Year 5: WPS5, TEC2.

SIGNIFICANCE

If teacher education is to bring about changes in teachers' practice, it requires insights about the teaching-learning relationship, motivation for making changes, techniques for putting the insights to work, and support for embedding the techniques in practice (Sims et al. 2023). The graduate school AMTLR aims to contribute to a strengthened research base for teacher education in two ways. First, the individual graduate projects will focus on building stronger support for insights regarding how specific teaching designs affect specific learning processes and outcomes in mathematics. The results from these projects will produce new insights about which teaching practices support students' mathematics learning, as well as how and why they do so—crucial knowledge for teacher education. The doctoral students will present their projects to teacher educators at the universities at mid- and post-conferences. Second, the AMTLR doctors will have robust knowledge and skill in how to plan, conduct and critically evaluate research regarding the relationship between mathematics teaching and learning. Therefore, they will be well-equipped to continue to contribute to the research base of mathematics teacher education by their own research and by critically evaluating and selecting insights and techniques for inclusion in the content of teacher education courses.

NATIONAL COORDINATION

AMTLR is a joint programme organised by three universities: UMU, LNU and MDU. It is hosted by UMU, where coordinator ass. prof. Anna Ida Säfström, Department of Science and Mathematics Education and UMERC, is employed. The AMTLR board consists of three faculty members from the three partner universities: Prof. Torulf Palm, Department of Science and Mathematics Education and UMERC, UMU, Prof. Per Nilsson, Department of Mathematics, LNU, and Prof. Andreas Ryve, School of Education, Culture and Communication, MDU. Two doctoral student representatives will also be part of the board.

Anna Ida Säfström is expert in students' mathematical reasoning—a key mathematics learning process. She is currently PI for a design research project focusing the design of teaching in Grades 1–3 for developing sustainable knowledge about arithmetic and engaged in another design research project focusing the design of tasks and teacher support in mathematics in Grades 4–6, both funded by SIER. She has experience of supervision and teaching on advanced and doctoral level, as well as experience as coordinator of teacher education, and is the codirector of UMERC. **Torulf Palm** has vast experience in scientific leadership and research on the teaching-learning relationship. He has been the director of the Regional Centre for In-service Training based at UMU and currently serves as the director of UMERC. He also leads a research group focusing on formative assessment in mathematics education. He is currently leading two projects funded by SRC and Marcus and Amalia Wallenberg foundation. Both projects focus on the mathematics teaching-learning relationship by investigating the mechanisms underlying the effects of formative assessment on student achievement in mathematics in laboratory settings. He has led several large research projects focused on, e.g., the teaching-learning relationship in multi-year research and development projects, where the effects of formative assessment on student learning in authentic classrooms have been studied using RCTs, classroom observations, interviews, questionnaires, and mathematics tests. **Per Nilsson** is expert in educational design research. He is responsible for a recurring symposium on design research in mathematics education, which brings together international experts in the field. He has served as the scientific leader of several research and school development projects, coordinated a graduate school in mathematics education and been a member of the board for a graduate school in educational sciences. **Andreas Ryve** has been the PI for several research projects funded by the Swedish Research Council and has directed several collaborations with municipalities and agencies. He has also served as a board member for two graduate schools funded by SRC and was the Dean of Educational Studies at MDU 2017–2024.

Between them, the three universities are responsible for the quality of the programme as well as for administrative and practical issues. Policy decisions of AMTLR are made by the board, for instance, with regard to course syllabi. The coordinator is responsible for organising the board meetings and coordinating the organisation of joint seminars (WPSs) and conferences (TECs). This also includes conducting and summarising evaluations of courses, WPSs and TECs that forms the basis of development of later activities, and a final evaluative report of the whole graduate programme. The coordinator and the three board members are responsible for planning and running AMTLR activities at the respective universities, including courses and supervision, in collaboration with local management and staff.

INTERNATIONAL AND OTHER NATIONAL COLLABORATION

AMTLR is supported by an international advisory board consisting of Prof. Paul Cobb, Vanderbilt University, US, expert in improvement of quality of mathematics teaching; Prof. Despina Potari, University of Athens, Greece, expert in development of mathematics teaching and the role of context; Prof. Bert Jonsson, UMU, expert in educational psychology; and Prof. Gavin Brown, University of Auckland, Australia, expert in cross-cultural educational psychology. The advisory board will advise the AMTLR board and also contribute directly to the doctoral students' education and networking in courses, WPSs and conferences.

In addition, **UMERC** has research collaborations with researchers from eight countries, including with Prof. John Hattie at University of Melbourne, Australia, and Prof. Dylan Wiliam at University College London, England. Hattie specializes in effectiveness studies and meta-analyses, which may contribute to the study of the teaching-learning relationship. Wiliam also has strong focus on the effectiveness of teaching on student achievement. Both also have a strong interest in the professional development of teachers and teacher students and have made seminal contributions to the area of feedback and formative assessment. UMERC also collaborates with researchers in EPC and neuroscience at UMU. The research group in mathematics education at **LNU** participates in several national and international collaborations. These include projects on variation theory as an instructional design principle in mathematics education, partnerships focused on mathematics teacher education, intercultural comparisons of mathematics teaching, and a global network on early mathematics. Collaborating countries include Germany, Ireland, Norway, the United States, South Africa, Finland, Italy, Australia, New Zealand, and China. The research group in mathematics education at **MDU** collaborates with researchers from a variety of universities within Sweden and around the world. Examples of collaborations of particular relevance for this graduate school are those with Prof. Paul Cobb about how to conceptualize and develop high-quality mathematics teaching and Professor Jennifer Gore at University of Newcastle in Australia about adapting and testing an impactful Australian professional development program of classroom observations (Quality Teaching Rounds, QTR), in the Swedish context.

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